

Perturbations of Ambient Magnetic Field Resulted from a Ball Motion in a Conductive Liquid Half-Space

Vadim V. Surkov^{1, 2, *}, Valery M. Sorokin¹, and Alexey K. Yashchenko¹

Abstract—We theoretically analyze perturbations of ambient magnetic field due to electric currents caused by motion of a dielectric ball in a conductive fluid half-space. The approximate analytical solution of the problem has been derived for the case of arbitrary orientation of the ambient magnetic field and under the requirement that the fluid flow around the ball is laminar and potential in character. We examine spatiotemporal distribution of these perturbations and their dependence on both the depth and distance from the moving ball. Amplitudes of electromagnetic perturbations generated by the fluid flow around the ball have been compared with that resulting from gravity waves in the fluid.

1. INTRODUCTION

The motion of conducting fluids such as seawater gives rise to perturbations of the Earth magnetic field. The rationale of these perturbations is that conductive fluid flow generates electric currents, which in turn result in perturbation of ambient magnetic field. Different types of sea waves, such as surface waves excited by wind, ship and internal waves, tidal streams, and tsunami waves, can generate the geomagnetic perturbations (GMPs) [1, 2]. Similar mechanisms, i.e., the generation of GMPs, can be responsible for naturally occurring co-seismic electromagnetic effects associated with seismic wave propagation in solid geological media having conductivity, for example, in porous water-saturated rocks [3]. At the ionospheric D- and E-layer altitudes, the neutral winds and acoustic gravity waves give rise to entrainment of the ionospheric plasma particles with neutral molecules thereby exciting ionospheric currents and GMPs [4, 5].

Theoretical study of the GMPs produced by seawater motion is based on the joint analysis of combined set of hydrodynamic and quasi-stationary Maxwell's equations describing the fluid mass velocity, electromagnetic fields and current flowing in the conductive seawater. For all case studies of interest here the hydrodynamic pressure is much greater than the magnetic one so that we can neglect the magnetic pressure term in the hydrodynamic equations [1, 2]. This means that the fluid mass velocity distribution is independent of the electromagnetic field parameters.

In the case of infinite fluid space there are a few simple analytical hydrodynamic solutions describing the laminar fluid flow around an infinite cylinder, ball and ellipsoid [6]. In the case of fluid half-space the solution of hydrodynamic problem gets more complicated since there is a need to take into account the boundary conditions not only on the body surface but also on the free surface of a fluid [7, 8].

The complicated structure of the ship waves makes the analytical description of the GMPs more difficult [9, 10]. It is usually the case that numerical simulation is used to analyze such kind of problem [11–14]. The exact solution of the problem has been obtained by Surkov et al. [15] for the case of laminar fluid flow around a dielectric ball moving in infinite conductive fluid space. The perturbations of ambient magnetic field are shown to decrease with distance r from the ball as r^{-2} while the angular

Received 28 January 2018, Accepted 30 March 2018, Scheduled 12 April 2018

* Corresponding author: Vadim V. Surkov (surkovvadim@yandex.ru).

¹ Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences, Moscow, Troitsk, Russia. ² JSC Research-and-Production Corporation “Precision Systems and Instruments”, Moscow, Russia.

and spatial distributions of these perturbations depend essentially on the angle between the vector of ball velocity and the vector of ambient magnetic field.

In this paper, we generalize the model considered by Surkov et al. [15] for the case of ball motion in the conductive liquid half-space. The main goal of this study is to derive an analytical solution of the problem, to examine the effect of free fluid surface on spatial distribution of the electromagnetic perturbations, and to estimate the electromagnetic signal amplitude produced by the ball motion as a function of distance.

2. BASIC EQUATIONS

Consider a dielectric solid ball moving in the conductive liquid space immersed in the homogeneous magnetic field \mathbf{B}_0 . In what follows we study the perturbations of the ambient magnetic field caused by electric currents originated from the conducting fluid flow around the solid body. The ball velocity \mathbf{V}_0 is assumed constant. In a reference frame fixed to the ball, there will be a steady fluid flow around the ball. By making the crude assumption that the fluid viscosity is negligible, the flow pattern is treated as a laminar and potential fluid flow. Thus the fluid mass velocity \mathbf{V} can be expressed via the hydrodynamic potential Φ through $\mathbf{V} = \nabla\Phi - \mathbf{V}_0$. Here the function Φ must obey Laplace equation $\nabla^2\Phi = 0$. Consider first the solid body of spherical shape moving in unlimited liquid space. In such a case the potential are given by $\Phi_0 = -(\mathbf{p} \cdot \mathbf{r})/(4\pi r^3)$ where $\mathbf{p} = 2\pi R^3\mathbf{V}_0$, r is a distance from the ball center, and R denotes the ball radius [6]. It is pertinent to note that if the distance r is much greater than the typical body size, the potential will be of the same form even though the body has an arbitrary shape. In this case \mathbf{p} denotes the so-called hydrodynamic dipole moment which is codirected with the vector of body velocity \mathbf{V}_0 . The absolute value of this dipole moment is the same order of magnitude as the body volume multiplied by $|\mathbf{V}_0|$.

Now we consider a fluid of infinite depth bounded above by a free surface. The origin of the coordinate system is situated at the undisturbed fluid surface $z = 0$ while z axis is vertically upward and is perpendicular to this surface. Suppose that the submerged ball moves horizontally along the x axis at the depth $z = -h$; that is, parallel to the free surface. In a reference frame fixed to the ball, there will be the boundary condition at the free fluid surface given by [6]

$$\partial_{xx}^2\Phi + (g/V_0^2)\partial_z\Phi = 0 \quad (1)$$

where g is the free fall acceleration, and $\partial_x = \partial/\partial x$ and $\partial_z = \partial/\partial z$ denote partial derivatives with respect to x and z , correspondingly. In such a case, one should seek for the solution of the problem in the form: $\Phi = \Phi_0 + \Phi_1$, where Φ_0 is the potential of laminar flow around a body moving in an infinite fluid whereas Φ_1 is the potential required to satisfy the boundary conditions given by Equation (1).

Arzhannikov and Kotelnikov [8] have shown that the boundary condition at the free fluid surface can be simplified depending on Froude number $Fr = V_0/(gh)^{1/2}$. When the velocity is great; that is, under the requirement that $Fr \gg 1$, the boundary condition in Eq.(1) can be reduced to $\partial_{xx}^2\Phi = 0$. Conversely, if the velocity is small; that is, $Fr \ll 1$ then the above boundary condition takes on the form $\partial_z\Phi = 0$. In what follows we neglect the effect of surface waves on the curvature of the free fluid surface so that the above boundary conditions can be applied to the plane $z = 0$. In such a case the mirror-image method can be used in analogy to electrostatics. The hydrodynamic potential in the region $z < 0$ can be represented by the sum of the fields originated from the dipole situated at depth $z = -h$ and the dipole mirror-image located at depth $z = h$ symmetrically with respect to the plane $z = 0$ as shown in Figure 1. For the case $Fr \gg 1$ the directions of the dipole vector moments have to be opposite; otherwise these vectors are codirectional. The potentials of hydrodynamic flow which satisfy the Laplace equation and the above boundary conditions are given by

$$\Phi = \Phi_+ - \Phi_-, \quad \Phi_+ = \frac{p}{4\pi}\partial_x r_+^{-1}, \quad \Phi_- = \frac{p_{\pm}}{4\pi}\partial_x r_{\pm}^{-1}, \quad r_{\pm} = \left\{x^2 + y^2 + (h \pm z)^2\right\}^{1/2}. \quad (2)$$

As long as $Fr \gg 1$, one should substitute $p_+ = p$ into Equation (2). By contrast, if $Fr \ll 1$ then one should take $p_- = -p$.

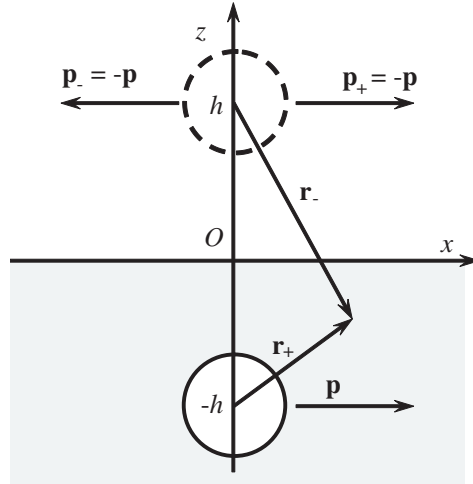


Figure 1. A schematic plot of a ball moving in conductive fluid and its “mirror image”. The vectors $\mathbf{p}_+ = \mathbf{p}$ and $\mathbf{p}_- = -\mathbf{p}$ of the ball hydrodynamic dipole moment correspond to small and large Froude numbers, correspondingly.

As far as the electrodynamic problem is concerned, the implication of this assumption is that we have neglected the electromagnetic perturbations resulting from the surface gravitational waves in the conductive fluid. The effect of the surface waves will be considered separately in Section 4.

Consider the homogeneous liquid half-space $z < 0$ with constant conductivity σ . In the reference frame fixed to the moving ball, the distribution of the fluid velocity \mathbf{V} , magnetic perturbations \mathbf{B} and electric perturbations \mathbf{E} are steady; that is, they are independent of time. To treat the electric and magnetic field in the conducting fluid, quasi-stationary Maxwell’s equations are needed, which, then reduce to the following ($B \ll B_0$)

$$D\nabla \times \mathbf{B} = \mathbf{E} + \mathbf{V} \times \mathbf{B}_0, \tag{3}$$

$$\nabla \times \mathbf{E} = 0, \tag{4}$$

where $D = (\mu_0\sigma)^{-1}$ is the coefficient of magnetic diffusion/viscosity, and μ_0 denotes the magnetic constant/magnetic permeability of free space.

The atmosphere is assumed to be an insulator which occupies the region $z > 0$. In the low-frequency range of interest the displacement current in the atmosphere can be neglected. We are thus left with the following set

$$\nabla^2 \mathbf{B} = 0, \quad \nabla^2 \mathbf{E} = 0. \tag{5}$$

Equations (3)–(5) should be supplemented by the proper boundary conditions. The normal component of the total current density has to be equal to zero at the dielectric ball surface and at the boundary between the atmosphere and fluid. In addition, the projections of magnetic perturbations and the tangential components of electric field must be continuous at these boundaries.

The exact solution of this problem has been recently obtained for the case of the ball motion in an infinite conductive fluid [15]. In such a case the problem is simplified since the fluid flow around the ball is described by only potential Φ_+ . If the origin of the coordinate system is situated in the center of the ball with radius R , this exact solution for the region outside the ball ($r > R$) can be rewritten by using the vector notation as follows

$$\mathbf{B}_1 = \frac{1}{8\pi D} \left(S(\mathbf{r}) \mathbf{r} + \frac{R^2}{5} \nabla S(\mathbf{r}) \right), \tag{6}$$

$$\mathbf{E}_1 = \nabla \frac{\mathbf{r} \cdot (\mathbf{p} \times \mathbf{B}_0)}{8\pi r^3}. \tag{7}$$

Here we made use of the following abbreviations

$$S(\mathbf{r}) = \frac{\mathbf{p} \cdot \mathbf{B}_0}{r^3} - \frac{3(\mathbf{p} \cdot \mathbf{r})(\mathbf{B}_0 \cdot \mathbf{r})}{r^5}. \tag{8}$$

where \mathbf{r} is the position-vector drawn from the center of the ball. Additionally, the constant electric field $\mathbf{E}_0 = \mathbf{V}_0 \times \mathbf{B}_0$ presented in this reference frame has been omitted from Equation (7).

Starting our study with the fluid half space $z < 0$ we seek for the solution of the problem in the form: $\mathbf{B}_f = \mathbf{B}_1 + \mathbf{B}_2$ and $\mathbf{E}_f = \mathbf{E}_1 + \mathbf{E}_2$, where \mathbf{B}_1 and \mathbf{E}_1 denote the solution in Eqs. (6)–(8) for the case of infinite fluid described by the hydrodynamic potential Φ_+ while \mathbf{B}_2 and \mathbf{E}_2 stand for unknown vector functions. The solution \mathbf{B}_1 and \mathbf{E}_1 must obey the requirement that normal component of total current density vanishes exactly on the ball surface. To verify this requirement, we first substitute Equation (6) for \mathbf{B}_1 into Maxwell equation $\nabla \times \mathbf{B}_1 = \mu_0 \mathbf{j}_1$. From here we find the corresponding total current density

$$\mathbf{j}_1 = -\frac{3\sigma}{8\pi r^5} \{(\mathbf{p} \times \mathbf{r})(\mathbf{B}_0 \cdot \mathbf{r}) + (\mathbf{B}_0 \times \mathbf{r})(\mathbf{p} \cdot \mathbf{r})\}, \quad (9)$$

whence it follows that the vector \mathbf{j}_1 is perpendicular to the position-vector \mathbf{r} everywhere, and thus the normal component of \mathbf{j}_1 equals zero at $r = R$; that is on the ball surface.

The unknown functions \mathbf{B}_2 and \mathbf{E}_2 must obey the initial set of Eqs. (3) and (4) in which the fluid velocity is given by $\mathbf{V} = \nabla \Phi_-$. Taking the curl of both sides of Equation (3) under the requirement that the fluid conductivity σ is constant, using Equation (4) and equation $\nabla \cdot \mathbf{B}_2 = \mathbf{0}$, we obtain that $D\nabla^2 \mathbf{B}_2 = -\nabla \times (\nabla \Phi_- \times \mathbf{B}_0)$. Rearranging the right side of this equation and taking into account constancy of vector \mathbf{B}_0 , we arrive at

$$D\nabla^2 \mathbf{B}_2 = -(\mathbf{B}_0 \cdot \nabla) \nabla \Phi_-. \quad (10)$$

We shall seek for the solution of Equation (10) in the form of Fourier transform with respect to x and y . Let k_x and k_y be the parameters of the Fourier transform while B_{0x} , B_{0y} and B_{0z} denote the projections of undisturbed magnetic field \mathbf{B}_0 onto coordinate axes. Applying Fourier transform to Equation (10) yields

$$\begin{aligned} D(b''_{x2} - k^2 b_{x2}) &= k_x(k_x B_{0x} + k_y B_{0y})f - ik_x B_{0z} f', \\ D(b''_{y2} - k^2 b_{y2}) &= k_y(k_x B_{0x} + k_y B_{0y})f - ik_y B_{0z} f', \\ D(b''_{z2} - k^2 b_{z2}) &= -i(k_x B_{0x} + k_y B_{0y})f' - k^2 B_{0z} f, \\ f &= -\frac{ip_{\pm} k_x}{4\pi k} \exp[k(z-h)], \quad k = (k_x^2 + k_y^2)^{1/2}, \end{aligned} \quad (11)$$

where the primes denote the derivatives with respect to z . The functions f , b_{x2} , b_{y2} , and b_{z2} stand for Fourier transform of the potential Φ_- and projections of the magnetic perturbation \mathbf{B}_2 , correspondingly. These functions depend on k_x , k_y and z . Notice that the latest equation in the set of Eqs. (11) has been rearranged using the equation $f'' = k^2 f$. The set of Eqs. (11) admits a solution subjected to the requirement that all the functions are finite when $z \rightarrow -\infty$

$$\begin{aligned} b_{x2} &= C_1 \exp(kz) + s_{\pm} ik_x z \exp[k(z-h)], \\ b_{y2} &= C_2 \exp(kz) + s_{\pm} ik_y z \exp[k(z-h)], \\ b_{z2} &= C_3 \exp(kz) + s_{\pm} kz \exp[k(z-h)], \end{aligned} \quad (12)$$

where $C_1 - C_3$ are arbitrary constants. Here we made use of the following abbreviation

$$s_{\pm} = \frac{k_x p_{\pm}}{8\pi D k^2} (k_x B_{0x} + k_y B_{0y} - ik B_{0z}). \quad (13)$$

The solution of the problem must obey the equation $\nabla \cdot \mathbf{B}_2 = \mathbf{0}$. The Fourier transform of this equation is given by

$$i(k_x b_{x2} + k_y b_{y2}) + b'_{z2} = 0. \quad (14)$$

Substituting Equations (12) for b_{x2} , b_{y2} , and b_{z2} into Equation (14), we obtain that the latter equation is valid under the following requirement

$$ik_x C_1 + ik_y C_2 + k C_3 = -s_{\pm} k \exp(-kh). \quad (15)$$

Let \mathbf{e}_2 be a Fourier transform of the electric field \mathbf{E}_2 . Applying a Fourier transform to Equation (3) we can relate \mathbf{e}_2 through the magnetic perturbation given by Equation (12)

$$\begin{aligned} e_{2x} &= D (ik_y b_{2z} - b'_{2y}) + ik_y B_{0z} f - B_{0y} f', \\ e_{2y} &= D (b'_{2x} - ik_x b_{2z}) + B_{0x} f' - ik_x B_{0z} f, \\ e_{2z} &= Di (k_x b_{2y} - k_y b_{2x}) + i (k_x B_{0y} - k_y B_{0x}) f. \end{aligned} \quad (16)$$

Since the atmosphere is assumed to be an insulator, the normal component of the total current density must equal zero at the boundary $z = -0$ between the fluid and the atmosphere. This means that the normal component of vector $\nabla \times (\mathbf{B}_1 + \mathbf{B}_2)$ equals to zero at $z = -0$. Applying Fourier transform to this boundary condition yields

$$k_x (b_{y1} + b_{y2}) = k_y (b_{x1} + b_{x2}). \quad (17)$$

The Fourier transforms b_{x1} , b_{y1} and b_{z1} of the vector \mathbf{B}_1 projections on the surface $z = 0$ are found in Appendix A.

It should be noted that in the approach treated here the boundary conditions on the ball surface are not valid for the components \mathbf{B}_2 , \mathbf{E}_2 and for their Fourier transforms. However, if the submergence depth of the ball is much greater than the ball size, then in the vicinity of the ball the absolute values of \mathbf{B}_2 and \mathbf{E}_2 are much smaller than $|\mathbf{B}_1|$ and $|\mathbf{E}_1|$, correspondingly. We shall therefore assume that $h \gg R$. This means, in particular, that the boundary condition for the total current density on the ball surface will be approximately valid. Nevertheless, in this model the boundary conditions on the interface $z = 0$ are valid exactly.

In order to find the electromagnetic perturbations in the atmosphere, we apply a Fourier transform to Equation (5). The results are given by $\mathbf{b}''_A - k^2 \mathbf{b}_A = 0$ and $\mathbf{e}''_A - k^2 \mathbf{e}_A = 0$. Taking into account that the solution of the problem has to be finite as $z \rightarrow \infty$, we come to

$$\mathbf{b}_A = \mathbf{N} \exp(-kz), \quad \mathbf{e}_A = \mathbf{M} \exp(-kz), \quad (18)$$

where the components of vectors $\mathbf{N} = (N_1, N_2, N_3)$ and $\mathbf{M} = (M_1, M_2, M_3)$ are arbitrary constants. Substituting Equation (18) for the magnetic perturbations \mathbf{b}_A into Equation (14), we obtain that these constants obey the following condition

$$ik_x N_1 + ik_y N_2 = k N_3. \quad (19)$$

The solution of the problem must be continuous at the boundary $z = 0$. In order to satisfy this requirement we need to match the solutions for the fluid $\mathbf{b}_f = \mathbf{b}_1 + \mathbf{b}_2$ and for the atmosphere \mathbf{b}_A at $z = 0$. As a result we come to the following set of equations for the unknown coefficients

$$N_1 = C_1 + b_{x1}, \quad N_2 = C_2 + b_{y1}, \quad N_3 = C_3 + b_{z1}. \quad (20)$$

One can find the unknown coefficients by solving the set of algebraic Equations (15), (17), (19) and (20) and thus to construct the Fourier representation of solution to the problem. In a similar fashion one may find the unknown coefficients M_1 , M_2 and M_3 for the electric field in the atmosphere. To do this requires the continuity of e_x and e_y at the boundary between fluid medium and the atmosphere. Additionally the requirement $\nabla \cdot \mathbf{E} = 0$ at $z \geq 0$ is needed that leads to condition similar to Equation (14). The solution and detailed calculations for both cases of small and large Froude numbers are found in Appendix A. The components of electromagnetic perturbations in the atmosphere, \mathbf{b}_A and \mathbf{e}_A , are given by Equations (A7), (A10) and (A11). The components \mathbf{b}_2 and \mathbf{e}_2 for the fluid medium are given by Equations (A8), (A9) and (A12).

3. A SPATIAL REPRESENTATION OF THE ELECTROMAGNETIC PERTURBATIONS

Spatial distribution of the electromagnetic perturbations can be derived using an inverse Fourier transform over parameters k_x and k_y . The corresponding calculations in more detail are found in Appendix B. At first consider the case of large Froude number ($Fr \gg 1$), which corresponds to the

ball motion at large speed or at small depth. Applying inverse Fourier transform to Equation (A7), we obtain the electromagnetic perturbations in the atmosphere:

$$\mathbf{B}_A = -\frac{p}{8\pi D} \nabla \left\{ h (\mathbf{B}_0 \cdot \nabla) G_{1+} + B_{0z} G_{1+} - \frac{R^2}{5} (\mathbf{B}_0 \cdot \nabla) \frac{x}{r_+^3} \right\}. \quad (21)$$

$$\begin{aligned} E_{Ax} &= -\frac{p}{8\pi} \left\{ (\mathbf{B}_0 \times \nabla_\perp)_z \partial_x G_{1+} - \frac{9x}{r_+^5} [y B_{0z} + (h+z) B_{0y}] \right\}, \\ E_{Ay} &= \frac{p}{8\pi} \left\{ -(\mathbf{B}_0 \times \nabla_\perp)_z \partial_y G_{1+} + \frac{3}{r_+^5} [3x(h+z) B_{0x} + (3x^2 - r_+^2) B_{0z}] \right\}, \\ E_{Az} &= \frac{p}{8\pi r_+^5} \{-3xy B_{0x} + (3x^2 - r_+^2) B_{0y}\}, \end{aligned} \quad (22)$$

where $\nabla_\perp = \hat{\mathbf{x}}\partial_x + \hat{\mathbf{y}}\partial_y$ stands for the operator of transverse gradient while $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ denote unit vectors of coordinate axes. Here and below we use the following designations:

$$G_{1\pm} = \frac{x}{r_\pm (r_\pm + h \pm z)}, \quad G_{2\pm} = \frac{1}{r_\pm (x^2 + y^2)} \left\{ \frac{(h \pm z)(x^2 - y^2)}{r_\pm + h \pm z} + y^2 \right\}. \quad (23)$$

In a similar fashion we obtain the components \mathbf{B}_2 and \mathbf{E}_2 of the electromagnetic perturbations in the fluid

$$\begin{aligned} B_{2x} &= \frac{p}{8\pi D} \left\{ (\mathbf{B}_0 \cdot \nabla_\perp) G_{2-} + \frac{2xB_{0x} + yB_{0y} + (h-z)B_{0z}}{r_-^3} - B_{0z}\partial_x G_{1-} - m_x \right\}, \\ B_{2y} &= \frac{p}{8\pi D} \{ (\mathbf{B}_0 \times \nabla_\perp)_z G_{2-} - B_{0z}\partial_y G_{1-} - m_y \}, \quad B_{2z} = \frac{p}{8\pi D} \left\{ \frac{x B_{0z}}{r_-^3} - m_z \right\}, \\ E_{2x} &= -\frac{p}{8\pi} \left\{ (\mathbf{B}_0 \times \nabla_\perp)_z \partial_x G_{1-} + \frac{6x}{r_-^5} [(h-z) B_{0y} + 2y B_{0z}] \right\}, \\ E_{2y} &= \frac{p}{8\pi} \left\{ -(\mathbf{B}_0 \times \nabla_\perp)_z \partial_y G_{1-} + \frac{1}{r_-^5} [3(h-z)(y B_{0y} + 3x B_{0x}) + (9x^2 - 3y^2 - 2r_-^2) B_{0z}] \right\}, \\ E_{2z} &= \frac{p}{8\pi r_-^5} \{-12xy B_{0x} + (2r_-^2 - 6x^2 + 3y^2) B_{0y} + 3y(h-z) B_{0z}\}, \end{aligned} \quad (24)$$

where m_x , m_y and m_z are projections of the vector $\mathbf{m} = z\nabla(\mathbf{B}_0 \cdot \nabla)G_{1-}$ while the functions G_{1-} and G_{2-} are given by Equation (23). The total electromagnetic perturbations in the fluid is a sum of two summands $\mathbf{B}_f = \mathbf{B}_1 + \mathbf{B}_2$ and $\mathbf{E}_f = \mathbf{E}_1 + \mathbf{E}_2$. The terms \mathbf{B}_1 and \mathbf{E}_1 are given by Equations (6) and (7) where \mathbf{r} must be replaced by \mathbf{r}_+ . It should be noted that the summands contained the factor R^2 do not enter Equation (24) for \mathbf{B}_2 and \mathbf{E}_2 . However the electromagnetic perturbations in the fluid medium are dependent on the ball radius since the factor R^2 is present in the expressions for \mathbf{B}_1 and \mathbf{E}_1 .

It follows from equations for \mathbf{E}_A and \mathbf{E}_f that the vertical projection of the electric field has a discontinuity at the surface $z = 0$ because of the generation of surface electric charge at the boundary between the media.

Similar reasoning shows that in the case of $Fr \ll 1$ the electromagnetic perturbations in the atmosphere are given by ($z > 0$):

$$\begin{aligned} \mathbf{B}_A &= -\frac{p}{8\pi D} \left\{ \nabla \left[h (\mathbf{B}_0 \cdot \nabla) G_{1+} - \frac{R^2}{5} (\mathbf{B}_0 \cdot \nabla) \frac{x}{r_+^3} \right] - \hat{\mathbf{x}} (\mathbf{B}_0 \cdot \nabla_\perp) G_{2+} - \right. \\ &\quad \left. - \hat{\mathbf{y}} \left[(\mathbf{B}_0 \times \nabla_\perp)_z G_{2+} - \frac{x B_{0y}}{r_+^3} \right] + \hat{\mathbf{z}} (\mathbf{B}_0 \cdot \nabla_\perp) G_{1+} \right\}, \\ E_{Ax} &= -\frac{3p}{8\pi} \left\{ (\mathbf{B}_0 \cdot \nabla_\perp) \partial_y G_{1+} - \frac{3xy}{r_+^5} B_{0z} \right\}, \quad E_{Az} = 0, \\ E_{Ay} &= \frac{3p}{8\pi} \left\{ (\mathbf{B}_0 \cdot \nabla_\perp) \partial_x G_{1+} + \frac{1}{r_+^3} \left(1 - \frac{3x^2}{r_+^2} \right) B_{0z} \right\}, \end{aligned} \quad (25)$$

while the projections of the vectors \mathbf{B}_2 and \mathbf{E}_2 onto coordinate axes can be written as

$$\begin{aligned}
 B_{2x} &= \frac{p}{8\pi D} \left\{ 2(\mathbf{B}_0 \nabla_{\perp}) G_{2-} + \frac{2xB_{0x} + yB_{0y} + (h-z)B_{0z}}{r_{-}^3} + m_x \right\}, \\
 B_{2y} &= \frac{p}{8\pi D} \left\{ 2(\mathbf{B}_0 \times \nabla_{\perp})_z G_{2-} - B_{0y} \frac{x}{r_{-}^3} + m_y \right\}, \quad B_{2z} = \frac{p}{8\pi D} \left\{ -(\mathbf{B}_0 \nabla_{\perp}) G_{1-} + m_z \right\}, \\
 E_{2x} &= \frac{3p}{8\pi} \left\{ -(\mathbf{B}_0 \cdot \nabla_{\perp}) \partial_y G_{1-} + \frac{x}{r_{-}^5} [(h-z)B_{0y} + 2yB_{0z}] \right\}, \\
 E_{2y} &= \frac{p}{8\pi} \left\{ 3(\mathbf{B}_0 \cdot \nabla_{\perp}) \partial_x G_{1-} + \frac{3y(h-z)}{r_{-}^5} B_{0y} + \frac{1}{r_{-}^3} \left[1 + \frac{3\{(h-z)^2 - 2x^2\}}{r_{-}^2} \right] B_{0z} \right\}, \\
 E_{2z} &= \frac{p}{8\pi r_{-}^3} \left\{ 2 \left[\frac{3x(x+y)}{r_{-}^2} - 1 \right] B_{0x} - \left(1 + \frac{6xy}{r_{-}^2} \right) B_{0y} + \frac{3y(h-z)}{r_{-}^2} B_{0z} \right\}.
 \end{aligned} \tag{26}$$

As before the total field in the fluid ($z < 0$) is a sum of two summands; that is $\mathbf{B}_f = \mathbf{B}_1 + \mathbf{B}_2$ and $\mathbf{E}_f = \mathbf{E}_1 + \mathbf{E}_2$.

The equations derived above can be simplified at far distances from the ball. For example, if $r_+ \gg R$, h and $Fr \gg 1$, then the magnetic perturbation in the atmosphere reduces to

$$\begin{aligned}
 B_{Ax} &\approx -\frac{pB_{0z}}{8\pi D r_+ (r_+ + h + z)} \left\{ 1 - \frac{x^2 (2r_+ + h + z)}{r_+^2 (r_+ + h + z)} \right\}, \\
 B_{Ay} &\approx \frac{pB_{0z} (2r_+ + h + z)}{8\pi D r^3 (r_+ + h + z)}, \quad B_{Az} \approx \frac{pB_{0z} x}{8\pi D r^3}.
 \end{aligned} \tag{27}$$

In the case of $Fr \ll 1$ the formula for \mathbf{B}_A is complicated even though the large distance approximation is used. However, these formulas can be simplified under additional requirement $r \gg z$; that is, for small altitudes in the atmosphere

$$\begin{aligned}
 B_{Ax} &\approx \frac{py}{8\pi D r^3} \left\{ 2B_{0y} - \frac{3y}{r^2} (xB_{0x} + yB_{0y}) \right\}, \\
 B_{Ay} &\approx \frac{p}{8\pi D r^3} \left\{ y \left[2B_{0x} + \frac{3y}{r^2} (xB_{0y} - yB_{0x}) \right] - xB_{0y} \right\}, \\
 B_{Az} &\approx \frac{p}{8\pi D r^2} \left\{ \frac{2xy}{r^2} B_{0y} - \left(1 - \frac{2x^2}{r^2} \right) B_{0x} \right\}.
 \end{aligned} \tag{28}$$

The main property of all these far-field approximations is that the amplitude of magnetic perturbations decreases with distance inversely proportional to the distance squared. By contrast, it follows from Equations (22), (24) and (26) that the amplitude of electric field decreases with distance as r^{-3} independently of the Froude number; that is, the electric field falls off more rapidly with distance than does the magnetic field. This means that at a distance well removed from the ball the electromagnetic perturbations are not governed by the magnetic dipole approximation

The same conclusion has been recently reached by Surkov et al. [15] for the case of a ball motion in an infinite conducting fluid space under assumption of laminar potential fluid flow around the ball. Some insight into this fact can be achieved by estimating the effective magnetic dipole moment of the induction currents $\mathbf{j}_{\text{eff}} = \sigma(\mathbf{V} \times \mathbf{B}_0)$ in the fluid conductive medium. Integrating the function $0.5(\mathbf{r} \times \mathbf{j}_{\text{eff}})$ over the region with radius r gives the magnetic moment which tends to infinity as $r \rightarrow \infty$ [15]. This means that the typical magneto-dipole asymptotic $B \propto r^{-3}$ cannot be derived from this model. However, away from the source the actual asymptotic law of the electromagnetic perturbations can differ from that derived in this model due to the vortex motion and turbulence fluid flow or because of the presence of the fluid bottom.

In order to illustrate the basic features of the solutions in Eqs. (21)–(26), the numerical calculations were carried out. In all cases treated below, the ball is assumed to move along x -axis at the depth $h = 5R$. Let α , β and γ be the angles between the vector of ambient magnetic field \mathbf{B}_0 and the axes

x , y and z , correspondingly. Consider first the case of horizontal magnetic field \mathbf{B}_0 ($\gamma = \pi/2$), which may be important to practice as the body moves in Equatorial Water. The dimensionless dependence of the horizontal magnetic perturbation B_h/B_* taken on the fluid free surface ($z = 0$) as a function of coordinates x/R and y/R are displayed in Figure 2 for the case of $Fr \gg 1$. Here we made use the following abbreviations: $B_h = (B_x^2 + B_y^2)^{1/2}$ and $B_* = pB_0/(8\pi DR^2)$. The field distributions shown in this figure correspond to different angles between the directions of the ball motion and the vector \mathbf{B}_0 . As seen from Figure 2, the magnetic perturbations demonstrate a peculiar kind of “directional selectivity” since the diagrams shown in this figure can have two or four directional lobes depending on the angle α . In other words, there are two or four directions in which the geomagnetic perturbations reach maximum/minimum values and these directions depend on the angle α .

Now suppose that the body moves along a geomagnetic meridian. In such a case the undisturbed magnetic field \mathbf{B}_0 is situated in the vertical plane x, z ($\beta = \pi/2$). Figure 3 shows the results of model calculations of the magnetic perturbations in vertical plane $y = 0$ for the case of $Fr \gg 1$. Plotted is dimensionless vertical component B_z/B_* versus coordinates x/R and z/R for different inclination angles α of the ambient magnetic field \mathbf{B}_0 . It is obvious from Figure 3 that these field distributions are more complicated than that shown in Figure 2 since there are four or six directional lobes depending on the inclination angle α .

The dimensionless field projections in the atmosphere as functions of x/R or y/R at altitudes $z = 0$ and $z = 2R$ are shown in Figures 4–7 with lines 1 and 2 (or 1' and 2'), correspondingly. The field components are normalized by the values $B_* = pB_0/(8\pi DR^2)$ and $E_* = pB_0/(8\pi R^3)$. The solid lines 1 and 2 correspond to the case $Fr \gg 1$ while the inverse case of small Froude number is shown with dotted lines 1' and 2'.

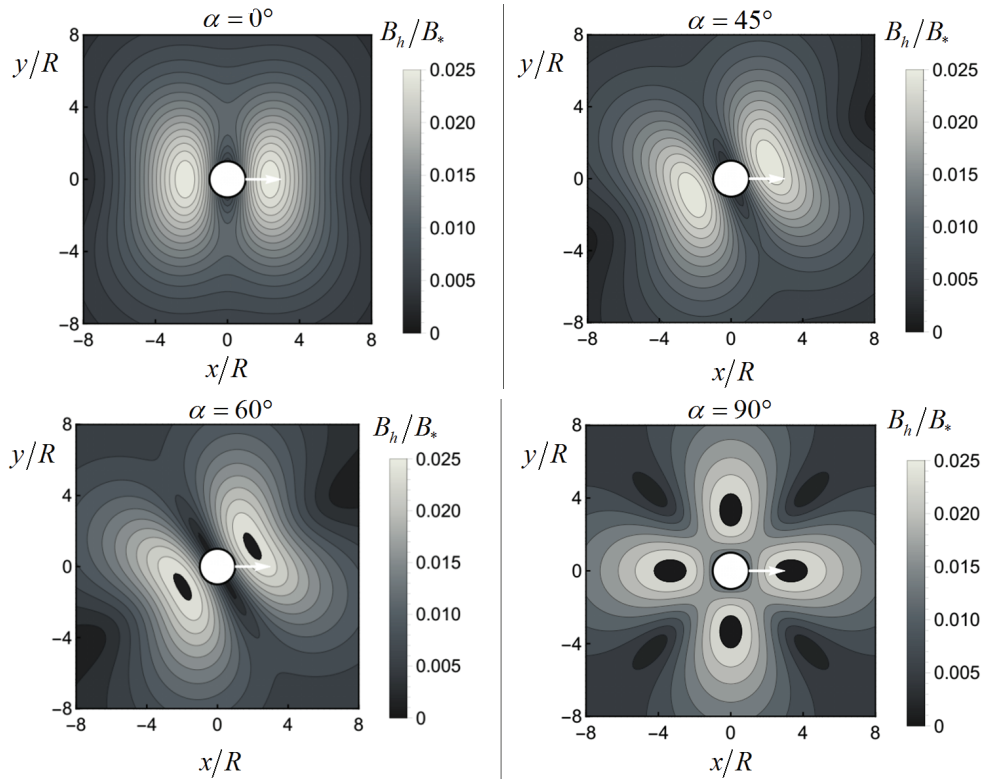


Figure 2. A model calculation of the horizontal components of magnetic perturbations on the fluid surface as a function of coordinates x and y for the case of $Fr \gg 1$. The ball moves along x -axis at the depth $h = 5R$ in horizontal magnetic field \mathbf{B}_0 . The pictures shown in this figure correspond to angles $\alpha = 0^\circ, 45^\circ, 60^\circ$ and 90° between the ball velocity and the vector \mathbf{B}_0 .

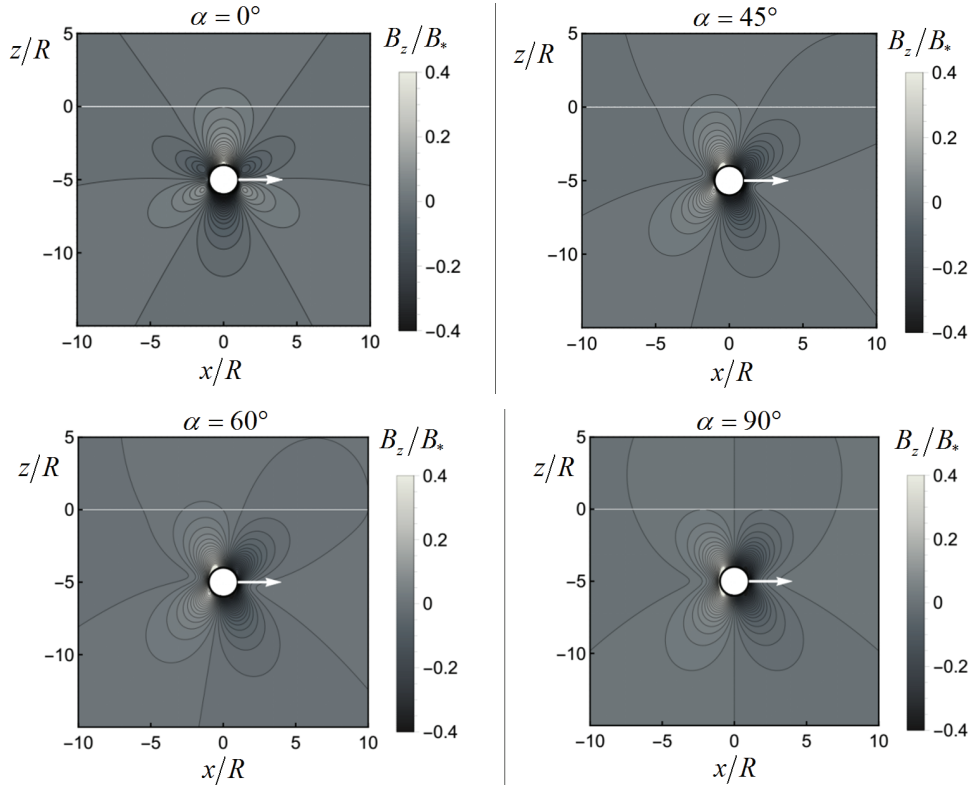


Figure 3. A model calculation of the vertical component of the magnetic perturbations in vertical plane $y = 0$ as a function of coordinates x and z for the case of $Fr \gg 1$. The ball moves along x -axis at the depth $h = 5R$ in the magnetic field \mathbf{B}_0 which makes the angles $\alpha = 0^\circ, 45^\circ, 60^\circ$ and 90° with respect to axis x .

In Figures 4 and 5, we plot the electromagnetic perturbations which can be excited at the magnetic equator. In this case the ambient magnetic field \mathbf{B}_0 is assumed to be horizontal and parallel to axis x ; that is, the vector \mathbf{B}_0 and the ball velocity are co-directed. Figure 4 shows the electromagnetic perturbations taken along the x -axis as a function of x/R . In this case only three components B_x, B_z and E_y are nonzero values, so only these components are shown in Figures 4(a), 4(b) and 4(c), correspondingly. As seen from these figures, the shapes of magnetic perturbations for the case of $Fr \gg 1$ are closely analogous to that for the inverse case of $Fr \ll 1$ though they are different in amplitude. In these extreme cases, on the contrary, the electric field components can have the similar shapes but different signs. The component B_z reaches a peak value just above the ball location whereas the absolute values of B_x and E_y tends to maximize behind of and in front of the ball center at normalized distances approximately equal to $x/R = 2.5$. If $Fr \gg 1$, the perturbation amplitudes are approximately two or three times as much again as that for the inverse case of $Fr \ll 1$. It is worth noting that in all cases the amplitudes of perturbations decrease rapidly with distance from the ball.

The same as in Figure 4 but in transverse direction are shown in Figures 5(a), (b), (c). Plotted are the nonzero components (a) B_y , (b) B_z and (c) E_x taken at $x = 0$ as functions of y/R . This figure exhibits similar features; that is, the maximum of B_z is situated above the ball location while the peaks of absolute values of B_y and E_x are at the normalized distances $y/R \approx 2.5 \div 4$ from the ball center. The additional peculiarity is that the electric perturbations are greater for the case of $Fr \ll 1$.

If the ball moves along the magnetic parallel; that is, perpendicular to the horizontal magnetic field \mathbf{B}_0 , then other three components will be nonzero: B_y, E_x and E_z . The numerical simulation has shown that the dependences of these components on coordinates are largely the same as that shown in Figures 4 and 5 and thus they are not displayed in this paper.

Figures 6 and 7 illustrate the case of vertical magnetic field \mathbf{B}_0 ($\alpha = \beta = \pi/2$ and $\gamma = 0$) which can

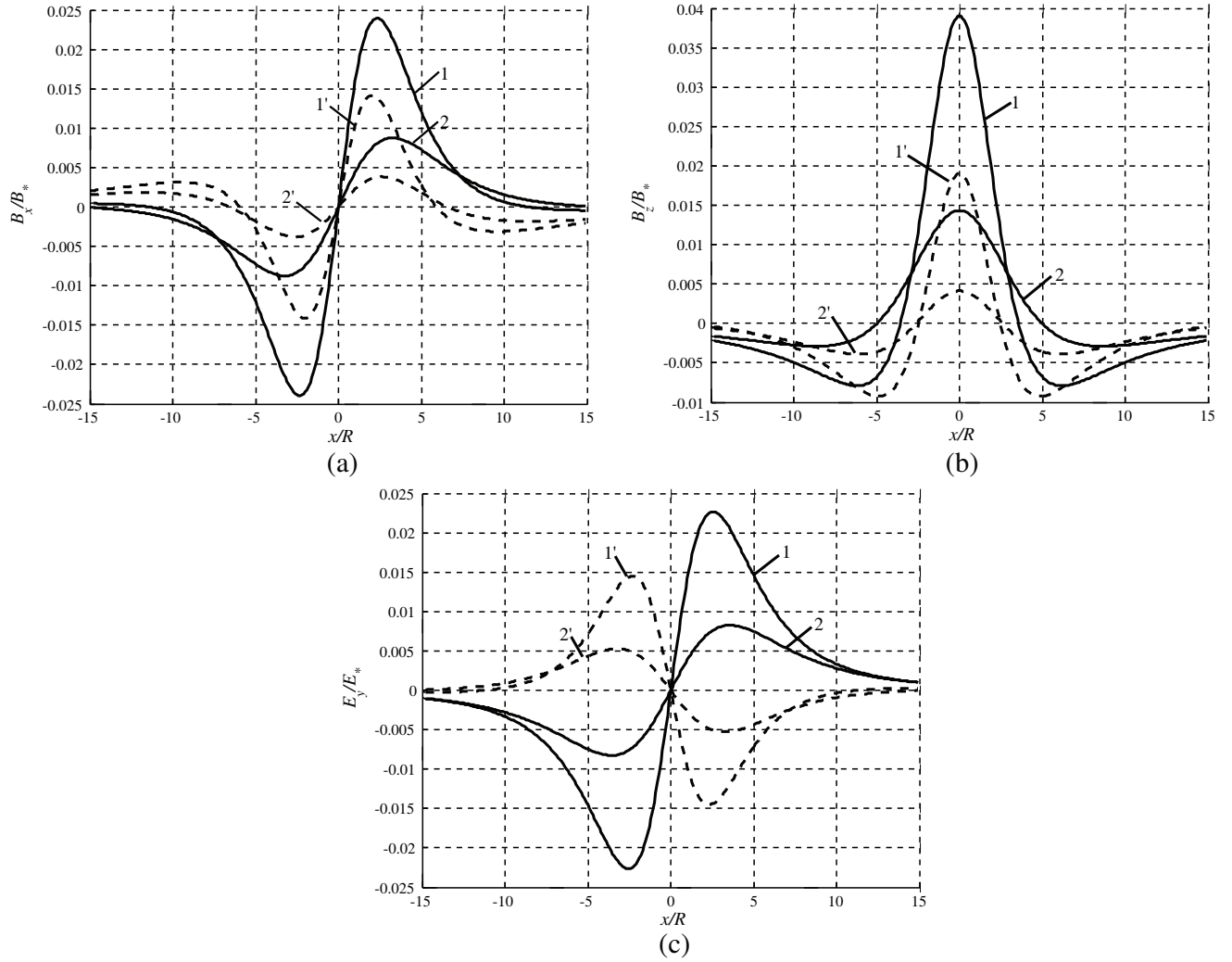


Figure 4. The normalized electromagnetic perturbations (a) B_x , (b) B_z and (c) E_y in the atmosphere calculated along x -axis versus dimensionless coordinate x/R . In the case of $Fr \gg 1$, these functions taken at the altitudes $z = 0$ and $z = 2R$ are shown in this figure with lines 1 and 2, correspondingly. The inverse case $Fr \ll 1$ is displayed with lines 1' and 2', correspondingly. The horizontal magnetic field \mathbf{B}_0 is parallel to both the ball velocity and to the axis x ($\alpha = 0$, $\beta = \gamma = \pi/2$).

be of great importance at polar latitudes. Figure 6 shows the result of calculations of the electromagnetic perturbations in the atmosphere taken along x -axis ($y = 0$) versus x/R . In Figures 6(a), (b) and (c), we plot the components B_x , B_z and E_y , correspondingly. In such a case other components of the electromagnetic perturbations on x -axis are equal to zero. It is interesting to note that the magnetic and electric amplitudes are greater if $Fr \ll 1$ in contrast to the case of horizontal magnetic field \mathbf{B}_0 . The same electromagnetic perturbations but in transverse direction; that is, along y -axes ($x = 0$) are shown in Figures 7(a) and (b). There are only two nonzero components: (a) B_x and (b) E_y . As seen from Figures 6 and 7, components B_x and E_y reach a peak value just above the ball whereas the absolute value of the vertical perturbation B_z has two maximums behind and in front of the ball location.

To take one example let us estimate the amplitude of GMPs caused by a ball moving in seawater. It follows from Equations (27) and (28) that away from the ball the amplitude of GMPs can be roughly estimated as $B_A \sim B_* R^2 / r^2 = \mu_0 \sigma p B_0 / 8\pi r^2$. We choose the following numerical values of the parameters: the conductivity of seawater $\sigma = 3 - 6$ S/m, the hydrodynamic dipole moment $p = 5 \cdot 10^4$ m⁴/s, induction of the Earth magnetic field $B_0 = 5 \cdot 10^{-5}$ T and the distance $r = 0.5 - 1$ km.

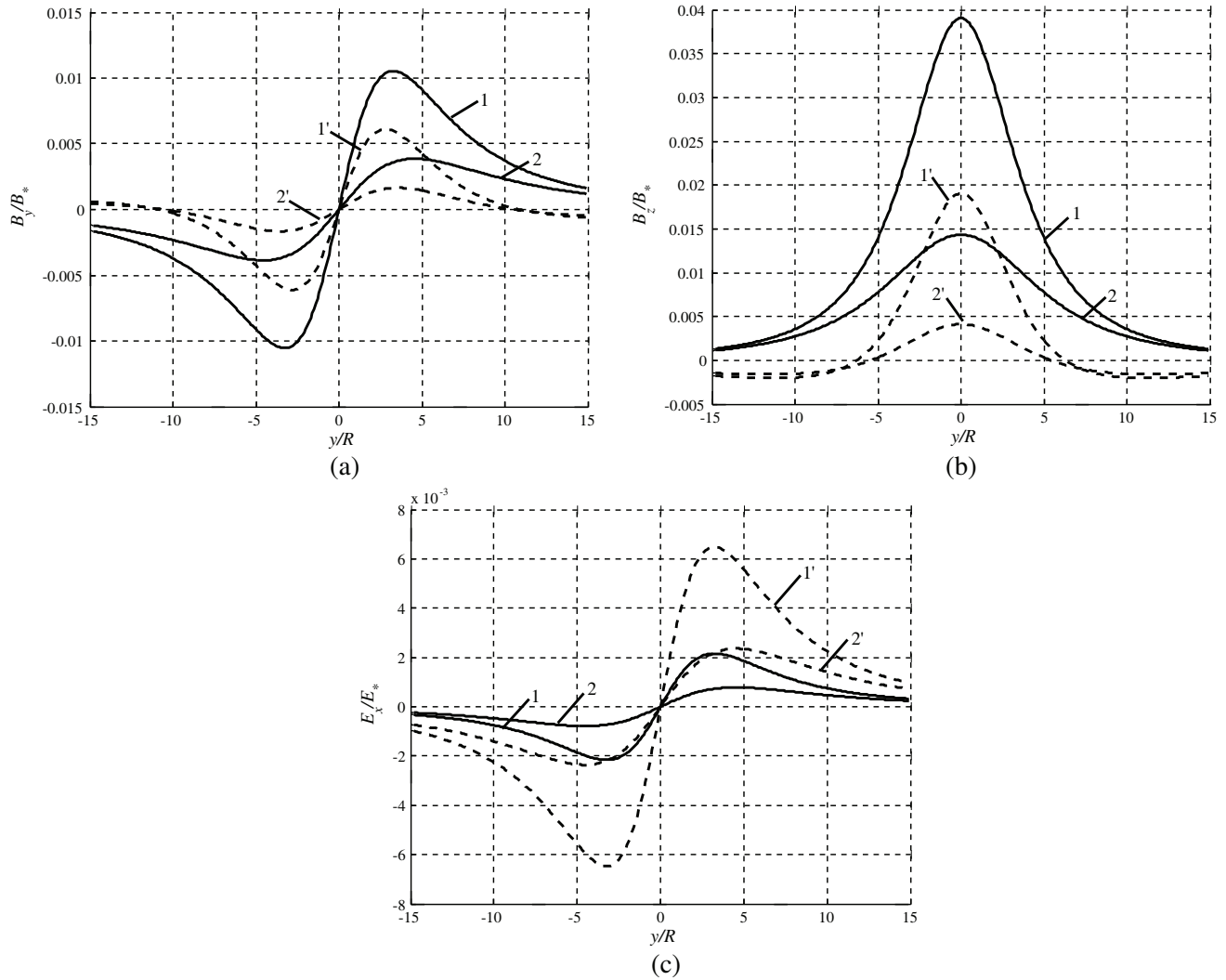


Figure 5. The normalized electromagnetic perturbations (a) B_y , (b) B_z and (c) E_x in the atmosphere calculated along y -axis versus dimensionless coordinate y/R . The same parameters and designations as that used in making Figure 4 have been used.

By substituting these parameters into above estimate, we obtain $B_A \sim 0.04 - 3$ pT. Analogously, we find a rough estimate of the electromagnetic field amplitude in the atmosphere $E_A \sim E_* R^3 / r^3 = pB_0 / 8\pi r^3 \approx 0.1 - 0.8$ nV/m.

4. ESTIMATE OF THE ELECTROMAGNETIC PERTURBATIONS CAUSED BY GRAVITY WAVES

In actual practice, the natural electromagnetic noises can mask the electromagnetic signal produced by the fluid flow around the ball. Here we do not concern the large scale fields generated by the ionospheric and magnetospheric current systems. The important source of the small scale GMPs and noises is the sea current systems generated by the wind-driven waves, surface gravity and internal waves and etc. In evaluating the signal-to-noise ratio, one should take into account that the amplitude of signal produced by the moving ball falls off rapidly with distance. The wave-driven noises can therefore dominate in the vicinity of the free fluid surface even if the mass velocity amplitude and wavelength are smaller than the amplitude and typical scale of the perturbations generated by fluid flow around the ball.

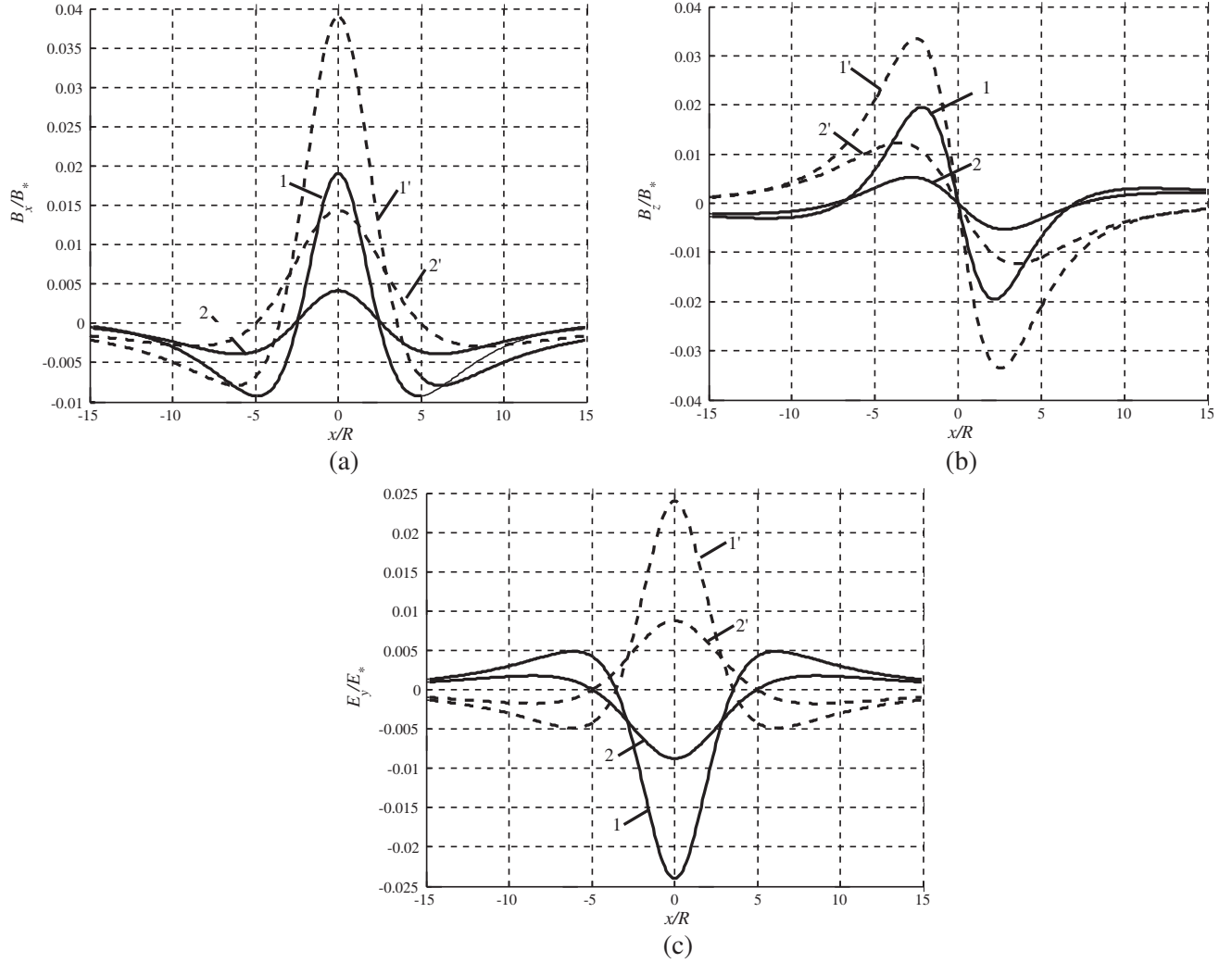


Figure 6. A model simulation of the electromagnetic perturbations (a) B_x , (b) B_z and (c) E_y in the atmosphere calculated along x -axis as functions of x/R . In the case of $Fr \gg 1$, these functions taken at the altitudes $z = 0$ and $z = 2R$ are shown in this figure with lines 1 and 2, correspondingly. The inverse case $Fr \ll 1$ is displayed with lines 1' and 2', correspondingly. The ambient magnetic field \mathbf{B}_0 is pointed upward ($\alpha = \beta = \pi/2$, $\gamma = 0$).

To estimate the contribution of the sea waves to the GMPs, we shall examine two-dimensional gravity wave propagating along horizontal x -axis in a perfect uniform fluid with constant conductivity σ . As before the origin of the coordinate system is situated on the plane surface $z = 0$ of undisturbed fluid while z -axis points vertically. As the bottom of the sea is situated at the depth $z = -H$, the solution of this hydrodynamic problem can be written in the form of quasi-plane harmonic wave ($z < 0$) [6]

$$\begin{aligned} V_z &= V_m \exp \{i(kx - \omega t)\} \sinh \{k(z + H)\} / \sinh(kH), \\ V_x &= iV_m \exp \{i(kx - \omega t)\} \cosh \{k(z + H)\} / \sinh(kH). \end{aligned} \quad (29)$$

Here V_z and V_x are the vertical and horizontal components of the fluid mass velocity, and V_m stands for the velocity amplitude. The circular frequency ω is related to the wave number k by the dispersion relation: $\omega^2 = gk \tanh(kH)$.

The small GMPs in the fluid layer ($-H < z < 0$) are described by Equation (3) as well as Maxwell equation $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$. The rocks situated below the seabed are treated as a uniform half-space ($z < -H$) with constant conductivity σ_g . Substituting σ_g instead of σ into Maxwell Equations (3),

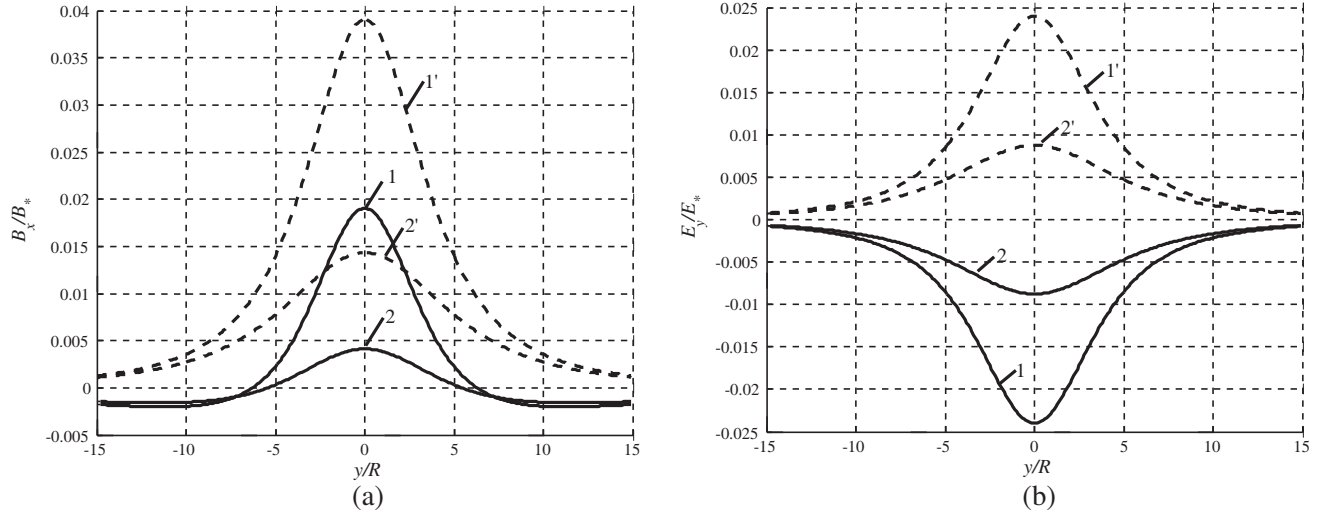


Figure 7. The normalized electromagnetic perturbations (a) B_x and (b) E_y in the atmosphere calculated along y -axis versus dimensionless coordinate y/R . The same parameters and designations as that used in making Figure 6 have been taken.

we obtain the equation for this region. The atmosphere ($z > 0$) is assumed to be the insulator where the conductivity and displacement currents can be ignored. The following equations: $\nabla^2 \mathbf{B} = 0$ and $\nabla^2 \mathbf{E} = 0$ are valid for this region. The components of \mathbf{B} , tangential projections of \mathbf{E} and normal component of electric current density must be continuous at all the boundaries. The solution of this problem presented in Appendix C shows that $B_y = 0$ in all the media. In the extreme case, when $H \rightarrow \infty$ the solution in Eq. (C5) for the atmosphere can be simplified to

$$B_{Az} = (iB_{0z} - B_{0x}) \frac{kV_m(\lambda - k)}{\omega(\lambda + k)} \exp(-kz), \quad B_{Ax} = -iB_{Az}, \quad (z > 0), \quad (30)$$

while the solution of Eq. (C6) for the fluid is reduced to

$$\begin{aligned} B_{fz} &= (iB_{0z} - B_{0x}) \frac{kV_m}{\omega} \left\{ \exp(kz) - \frac{2k}{\lambda + k} \exp(\lambda z) \right\}, \quad (z < 0); \\ B_{fx} &= (iB_{0z} - B_{0x}) \frac{kV_m}{\omega} \left\{ \exp(kz) - \frac{2\lambda}{\lambda + k} \exp(\lambda z) \right\}. \end{aligned} \quad (31)$$

where $\lambda^2 = k^2 - i\omega/D$ and $\text{Re}\lambda \geq 0$.

As is seen from Equation (30), the GMPs have the right-hand circular polarization in the atmosphere. It should also be noted that the GMP amplitude caused by the gravity waves decreases exponentially with height in the atmosphere and thus it falls off more rapidly with height than does the GMP due to the ball motion. To gain a better insight into this effect one should take into account that the wave-driven GMPs in the atmosphere are predominately due to the linear horizontal currents flowing along y -axis; that is, perpendicular to the vertical plane which contains the vector \mathbf{V} . The transversal spatial scale of these alternating-sign currents is about k^{-1} . Away from the fluid free surface the GMP caused by oppositely directed currents could compensate each other, which makes for the decrease of the GMPs with height. The typical vertical scale of the gravity wave-driven GMP in the atmosphere is of order k^{-1} ; that is, about several meters. This means that the atmospheric GMPs resulting from the fluid flow around the ball could dominate above tens meters.

The solution for electric field component E_y can be deduced from the equation $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ whence it follows that $E_y = \omega B_z/k$ where B_z is given by Equation (30) for the atmosphere or Equation (31) for the fluid medium. Other components of the electric field in the atmosphere take the form: $E_{Az} = iE_{Ax} = iV_m B_{0y} \exp(-kz)$ while for the fluid we obtain $E_{fx} = V_z B_{0y}$ and $E_{fz} = -V_x B_{0y}$ where the projections of the fluid mass velocity V_x and V_z are given by Equation (29).

When performing the electromagnetic field measurements on the fluid free surface, the observer moves at the fluid velocity \mathbf{V} together with equipment. In order to interpret these measurements correctly it is necessary to make the electromagnetic field transformation into the moving reference frame. In a nonrelativistic limit the observer will measure the fields $\mathbf{E}' \approx \mathbf{E} + \mathbf{V} \times \mathbf{B}_0$ and $\mathbf{B}' \approx \mathbf{B}$ [6]. In the extreme limit $H \rightarrow \infty$ the electromagnetic field components on the free surface ($z = 0$) are given by

$$\begin{aligned} B'_{Az} &= \frac{kV_m(\lambda - k)}{\omega(\lambda + k)}(iB_{0z} - B_{0x}), & B'_{Ax} &= -iB'_{Az}, \\ E'_{Ay} &= \frac{2V_mk}{\lambda + k}(B_{0x} - iB_{0z}), & E'_{Ax} &= E'_{Az} = 0. \end{aligned} \quad (32)$$

In order to simplify this solution we will now show that the inequality $k^2 \gg \omega/D$ is valid in the frequency range of interest. Using the dispersion relation for gravity waves we can reduce this inequality to the following $\omega \gg (\mu_0\sigma g^2)^{1/3}$. Taking the seawater conductivity $\sigma = 3-6$ S/m, we find that this requirement holds true for the frequencies $\nu = \omega/(2\pi) \gg (11 - 14)$ mHz whence it follows that $\lambda - k \approx -i\omega/(2Dk)$ and $\lambda + k \approx 2k$. Substituting these relationships into Equation (32), we obtain the approximate estimates

$$B'_{Az} = \frac{iV_m}{4Dk}(B_{0x} - iB_{0z}), \quad E'_{Ay} = V_m(B_{0x} - iB_{0z}). \quad (33)$$

Substituting $V_m = 0.1 - 1$ m/s, $\nu = 1$ Hz and the above-mentioned numerical parameters into this estimate yields $B'_A \sim 1 - 24$ pT and $E'_A \sim 0.5 - 5$ μ V/m. Assuming for the moment that this estimate can be applied to the low-frequency electromagnetic noise caused by the sea waves, we shall compare the noise amplitude with that produced by the ball motion on the fluid free surface. Simplifying and combining Equations (27), (28) and (33), we obtain that the magnetic signal-to-noise ratio near the free surface can exceed unity in the region restricted by the radius $r < r_* = \nu \{2\pi\rho/(gV_m)\}^{1/2}$. It seems likely that outside this region the low-frequency electromagnetic signal falls below the background noise level especially for the electric component. Using the above numerical parameters gives the value of the critical distance $r_* \approx (2 \div 6) \times 10^2$ m.

5. DISCUSSION AND CONCLUSIONS

The theoretical analysis has demonstrated that the spatial distribution of the electromagnetic field exhibits a number of the directional lobes which essentially depend on both the inclination angle of the ambient magnetic field \mathbf{B}_0 and the angle between \mathbf{B}_0 and the ball velocity vector. If the field \mathbf{B}_0 is horizontal, the magnetic perturbations are greater for the case of large Froude number than that for the inverse case of small Froude number. By contrast, if the field \mathbf{B}_0 is vertical, the magnetic perturbations are greater for the case of small Froude number. However, in both the cases the magnetic perturbations are the same order of magnitude and the magnetic field patterns are similar in character whereas the electric fields have opposite signs in these extreme cases.

Away from the moving ball the magnetic field amplitude B_{\max} was found to decrease with distance r as r^{-2} whereas the electric field amplitude has to decrease inversely proportional to the distance cubed. One may assume that such tendencies result from the model based on the laminar potential fluid flow around the ball. In such a case the usual magneto-dipole asymptotic law $B_{\max} \propto r^{-3}$ is invalid since the effective dipole moment of the induction fluid-driven current system tends to infinity. It appears that the magnetic dipole asymptotic may be deduced from more correct model which takes into account both the turbulence and viscosity of the actual fluid flowing around the ball.

In the vicinity of the ball the components of electromagnetic perturbations can reach a peak value both just above or below the ball and at the distances equal to several radii R from the ball center as displayed in Figures 4-7. Analysis of the analytical solution has shown that the terms contained the factor R^2 can be neglected for long distances from the ball. This means that the only hydrodynamic dipole moment \mathbf{p} and the vector \mathbf{B}_0 can serve as main parameters which determine the structure and asymptotic behaviour of the electromagnetic perturbations. One may assume that this feature holds true for the motion of submerged body with arbitrary shape at a distance well removed from the body.

The GMPs generated by sea waves give rise to the low-frequency electromagnetic noise in the atmosphere which makes difficulties for measuring of the signals produced by motion of the submerged body. Considering the gravity wave-driven GMPs as a noise source, we have found that the signal-to-noise ratio in the vicinity of free surface can exceed unity at only a short distance from the body. However, this tendency can be reversed with height since the GMPs caused by the sea waves decrease rapidly with height. This effect can be due to both the alternating-sign character and the random distribution of the currents generated by the gravity waves or other kind of waves in seawater.

APPENDIX A. A FOURIER TRANSFORM OF THE FIELDS FOR THE CASE OF INFINITE SPACE

First we shall find the Fourier transform of the electromagnetic perturbations \mathbf{B}_1 and \mathbf{E}_1 . In what follows a Fourier transform operator acting on arbitrary function $F(x, y)$ is represented by

$$\hat{\mathbf{I}}F = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y) \exp\{-i(k_x x + k_y y)\} dx dy. \quad (\text{A1})$$

Applying this operator to Equation (6) for \mathbf{B}_1 and performing integration by parts, we arrive at

$$\hat{\mathbf{I}}\mathbf{B}_1 = \frac{1}{8\pi D} \left[\hat{\mathbf{I}}\{S(\mathbf{r}_+) \mathbf{r}_+\} + \frac{R^2}{5} (ik_x \hat{\mathbf{x}} + ik_y \hat{\mathbf{y}} + \hat{\mathbf{z}} \partial_z) \hat{\mathbf{I}}\{S(\mathbf{r}_+)\} \right], \quad (\text{A2})$$

where

$$S(\mathbf{r}_+) = \frac{p}{r_+^3} \left(B_{0x} - \frac{3x\{xB_{0x} + yB_{0y} + h_+ B_{0z}\}}{r_+^2} \right). \quad (\text{A3})$$

Here we made use the following abbreviations $r_{\pm} = \{x^2 + y^2 + h_{\pm}^2\}^{1/2}$ and $h_{\pm} = h \pm z$. Substituting Equation (A3) for $S(\mathbf{r}_+)$ into Equation (A2), rearranging this equation and performing the integration, we obtain the Fourier transform of the magnetic perturbation \mathbf{B}_1 . In the case of $z = 0$ the projections of this transform are given by

$$\begin{aligned} b_{1x} &= P \exp(-kh) \left\{ \frac{ik_x B_{0x}}{k} (1 + Q_1) + \frac{ik_y B_{0y} Q_1}{k} - B_{0z} \left(1 - \frac{hk_x^2}{k} \right) + \frac{k_x R^2 Q}{5} \right\}, \\ b_{1y} &= \frac{Pk_x}{k} \exp(-kh) \left\{ -\frac{ik_y k_x B_{0x} \eta}{k^2} + iB_{0y} \left(1 - \frac{\eta k_y^2}{k^2} \right) + B_{0z} k_y h + \frac{kk_y R^2 Q}{5} \right\}, \\ b_{1z} &= P \exp(-kh) \left\{ k_x h \left(\frac{k_x B_{0x}}{k} + \frac{k_y B_{0y}}{k} + iB_{0z} \right) + \frac{ikk_x R^5 Q}{5} \right\}, \end{aligned} \quad (\text{A4})$$

where we have used the following auxiliary designations

$$P = \frac{p}{8\pi D}, \quad Q = \frac{i(k_x B_{0x} + k_y B_{0y})}{k} - B_{0z}, \quad Q_1 = 1 - \frac{\eta k_x^2}{k^2}, \quad \eta = 1 + kh. \quad (\text{A5})$$

Similarly, we can find a Fourier transform of the electric field \mathbf{E}_1 at $z = 0$ by acting the operator (A1) on Equation (7):

$$\mathbf{e}_1 = \frac{p}{8\pi} (ik_x \hat{\mathbf{x}} + ik_y \hat{\mathbf{y}} - k\hat{\mathbf{z}}) \left(B_{0y} + \frac{ik_y}{k} B_{0z} \right) \exp(-kh). \quad (\text{A6})$$

Much of what follows depends on Froude number. So, we shall therefore start our consideration with the case of $Fr \gg 1$. Substituting Equations (A4) for b_{1x} , b_{1y} , and b_{1z} into Equations (20) and rearranging these equations, we find the unknown coefficients N_1 , N_2 and N_3 , and then obtain the solution of the problem for the atmosphere ($z > 0$):

$$b_{Az} = iPw_+ \left\{ \frac{kk_x R^2 Q}{5} - Qk_x h - \frac{k_x B_{0z}}{k} \right\}, \quad b_{Ax,y} = -\frac{ik_{x,y} b_z}{k}. \quad (\text{A7})$$

Here, for brevity, the designation $w_{\pm} = \exp\{-k(h \pm z)\}$ has been used.

We shall now consider the region occupied by the conducting fluid. Substituting the coefficients N_1, N_2, N_3 and Equation (A4) for b_{1x}, b_{1y} , and b_{1z} into Equations (18)–(20) gives the undetermined coefficients $C_1 - C_3$. In order to find the magnetic perturbations \mathbf{b}_2 in the region $z < 0$, one should substitute these coefficients into Equation (12). As a result we get

$$\begin{aligned} b_{2x} &= -Pw_- \left\{ \frac{ik_x}{k} (g_1 + 1) B_{0x} + \frac{ik_y g_1}{k} B_{0y} - \left(\frac{k_y^2}{k^2} - \frac{zk_x^2}{k} \right) B_{0z} \right\}, \\ b_{2y} &= \frac{Pk_x}{k} w_- \left\{ \frac{ik_x k_y}{k^2} (1 - zk) B_{0x} - i \left(\frac{k_x^2}{k^2} + \frac{zk_y^2}{k} \right) B_{0y} - \frac{k_y}{k} (1 + zk) B_{0z} \right\}, \\ b_{2z} &= -\frac{Pk_x}{k} w_- \{ zk_x B_{0x} + zk_y B_{0y} + i(1 - zk) B_{0z} \}, \end{aligned} \quad (\text{A8})$$

where $g_1 = k_y^2/k^2 + zk_x^2/k$. It should be noted, that the terms containing the factor R^2 are cancelled and thus these terms are absent in Equation (A8).

In order to find the electric field \mathbf{e}_2 , we substitute Equation (A8) for b_{2x}, b_{2y} , and b_{2z} into Equation (16). In our case the function f contains the factor $p_+ = p$ so far as $Fr \gg 1$. Whence it follows that ($z < 0$):

$$\begin{aligned} e_{2x} &= -\frac{ipk_x w_-}{8\pi} \left\{ \frac{k_x k_y}{k^2} B_{0x} - \left(2 + \frac{k_x^2}{k^2} \right) B_{0y} + 4 \frac{ik_y}{k} B_{0z} \right\}, \\ e_{2y} &= -\frac{ipk w_-}{8\pi} \left\{ \frac{k_x}{k} \left(3 + \frac{k_y^2}{k^2} \right) B_{0x} + \frac{k_y^3}{k^3} B_{0y} + i \left(\frac{k_y^2 - 3k_x^2}{k^2} \right) B_{0z} \right\}, \\ e_{2z} &= \frac{pk w_-}{8\pi} \left\{ 4 \frac{k_x k_y}{k^2} B_{0x} + \frac{(3k_x^2 - k_y^2)}{k^2} B_{0y} - \frac{ik_y}{k} B_{0z} \right\}. \end{aligned} \quad (\text{A9})$$

In particular, Equation (A9) enables one to determine the horizontal components of the electric field on the fluid surface. Taking into account the boundary condition for the electric field at $z = 0$, one can find the constants M_1, M_2 and M_3 . Substituting these constants into Equation (18) gives the electric field in the atmosphere:

$$\begin{aligned} e_{Ax} &= \frac{ipk_x w_+}{8\pi} \left\{ -\frac{k_x k_y}{k^2} B_{0x} + \left(3 + \frac{k_x^2}{k^2} \right) B_{0y} - \frac{3ik_y}{k} B_{0z} \right\}, \\ e_{Ay} &= \frac{ipk_x w_+}{8\pi} \left\{ -\left(3 + \frac{k_y^2}{k^2} \right) B_{0x} + \frac{k_x k_y}{k^2} B_{0y} + \frac{3ik_x}{k} B_{0z} \right\}, \\ e_{Az} &= \frac{pk_x w_+}{2\pi k} (k_y B_{0x} - k_x B_{0y}). \end{aligned} \quad (\text{A10})$$

The case $Fr \ll 1$ can be treated in perfect analogy to that considered above. Similar reasoning shows that the electromagnetic field in the atmosphere is the following

$$\begin{aligned} b_{Az} &= iPw_+ \left\{ \frac{kk_x R^2 Q}{5} - Qk_x h + \frac{ik_x}{k^2} (k_x B_{0x} + k_y B_{0y}) \right\}, \quad b_{A_{x,y}} = -\frac{ik_{x,y} b_z}{k}, \\ e_{Ax} &= -\frac{3pk_x k_y w_+}{8\pi k^2} \{ ik_x B_{0x} + ik_y B_{0y} + kB_{0z} \}, \quad e_{Az} = 0, \\ e_{Ay} &= \frac{3pk_x^2 w_+}{8\pi k^2} \{ ik_x B_{0x} + ik_y B_{0y} + kB_{0z} \}. \end{aligned} \quad (\text{A11})$$

For the fluid medium we obtain

$$\begin{aligned}
 b_{2x} &= Pw_- \left\{ \frac{ik_x}{k} (g_2 - 1) B_{0x} + \frac{ik_y g_2}{k} B_{0y} + \left(1 + \frac{zk_x^2}{k} \right) B_{0z} \right\}, \\
 b_{2y} &= \frac{iPk_x w_-}{k} \left\{ \frac{k_x}{k} \left(\frac{2k_y}{k} + zk_y \right) B_{0x} + \left(\frac{2k_y^2}{k^2} + \frac{zk_y^2}{k} - 1 \right) B_{0y} - izk_y B_{0z} \right\}, \\
 b_{2z} &= \frac{Pk_x w_-}{k} \left\{ \frac{(zk - 1)}{k} (k_x B_{0x} + k_y B_{0y}) - izk B_{0z} \right\}, \\
 e_{2x} &= -\frac{pk_x w_-}{8\pi} \left\{ \frac{3ik_y}{k^2} (k_x B_{0x} + k_y B_{0y}) + iB_{0y} + \frac{2k_y}{k} B_{0z} \right\}, \\
 e_{2y} &= \frac{pkw_-}{8\pi} \left\{ \frac{3ik_x^3}{k^3} B_{0x} + \frac{ik_y}{k} \left(\frac{3k_x^2}{k^2} - 1 \right) B_{0y} + \left(1 + \frac{2k_x^2}{k^2} \right) B_{0z} \right\}, \\
 e_{2z} &= \frac{pkw_-}{8\pi} \left\{ -\frac{2k_x (k_x + k_y)}{k^2} B_{0x} + \frac{(k_x^2 - k_y^2 + 2k_x k_y)}{k^2} B_{0y} - \frac{ik_y}{k} B_{0z} \right\},
 \end{aligned} \tag{A12}$$

where the following designation is used: $g_2 = 2k_x^2/k^2 + zk_x^2/k - 1$.

APPENDIX B. AN INVERSE FOURIER TRANSFORM

In order to derive the spatial representation of the electromagnetic perturbations, we shall apply an inverse Fourier transform to Equations (A7)–(A12). In what follows the inverse transform operator acting on an arbitrary function $g(k_x, k_y)$ is represented by

$$\hat{\mathbf{A}}g = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(k_x, k_y) \exp\{i(k_x x + k_y y)\} dk_x dk_y. \tag{B1}$$

When performing integration in Equation (B1), it is suitable to introduce polar coordinates according to $k_x = k \cos \xi$ and $k_y = k \sin \xi$. In such a case the typical integrals resulting from the acting of operator in Eq. (B1) on functions in Eqs. (A7)–(A12) are as follows

$$G_{n\pm} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos^n \xi d\xi}{h_{\pm} - i(x \cos \xi + y \sin \xi)}, \quad n = 0, 1, 2. \tag{B2}$$

One should substitute $h_+ = h + z$ into Equation (B2) if $z \geq 0$ or $h_- = h - z$ if $z \leq 0$. Performing the integration over ξ yields

$$G_{0\pm} = \frac{1}{r_{\pm}}, \quad G_{1\pm} = \frac{x}{r_{\pm}(r_{\pm} + h_{\pm})}, \quad G_{2\pm} = \frac{1}{r_{\pm}(x^2 + y^2)} \left\{ \frac{h_{\pm}(x^2 - y^2)}{r_{\pm} + h_{\pm}} + y^2 \right\}. \tag{B3}$$

Notice that there is the relation between the functions $G_{0\pm}$ and $G_{1\pm}$, which follows from Equation (B3); that is $\partial_z G_{1\pm} = \pm \partial_x G_{0\pm}$.

Applying the operator in Eq. (B1) to Equations (A7) and (A10), using Equation (B2), and performing the integration give the electromagnetic perturbations in the atmosphere ($z > 0$) for the case of $Fr \gg 1$. We next rearrange these expressions and reduce them to a compact vector form given by Equations (21) and (22).

Similarly, applying the inverse Fourier transform to Equations (A8) and (A9), using Equation (B2), and performing the integration we come to Equation (24) for the spatial representation of the components \mathbf{B}_2 and \mathbf{E}_2 in the fluid half space $z < 0$.

Considering Equations (A11) and (A12), and making the same procedure one can obtain the solution for the case $Fr \ll 1$.

APPENDIX C. GMP GENERATED BY GRAVITY WAVES IN CONDUCTING FLUID

We seek for the solution of Equations (3) in the form:

$$\mathbf{B} = \mathbf{b}(z) \exp \{i(kx - \omega t)\}, \quad \mathbf{E} = \mathbf{e}(z) \exp \{i(kx - \omega t)\}, \quad (\text{C1})$$

Substituting relationships in Eqs. (29) and (C1) into Equation (3) and rearranging we obtain the following equation for vertical component of magnetic perturbations ($-h < z < 0$)

$$b_z'' - \lambda^2 b_z = \frac{ikV_m}{D \sinh(kh)} [iB_{0z} \cosh \{k(z+h)\} - B_{0x} \sinh \{k(z+h)\}], \quad (\text{C2})$$

where $\lambda^2 = k^2 - i\omega/D$. The solution of this equation is given by

$$b_z = C_1 \exp(\lambda z) + C_2 \exp(-\lambda z) + \frac{kV_m [iB_{0z} \cosh \{k(z+h)\} - B_{0x} \sinh \{k(z+h)\}]}{\omega \sinh(kh)}, \quad (\text{C3})$$

where C_1 and C_2 are undetermined coefficients. Below the seabed; that is in the region $z < -h$ the following equation takes place:

$$b_z'' - \lambda_g^2 b_z = 0, \quad (\text{C4})$$

where $\lambda_g^2 = k^2 - i\omega/D_g$ is complex wave number and $D_g = (\mu_0 \sigma_g)^{-1}$ stands for the coefficient of magnetic diffusion in the rocks contained in the seabed. The solution of Equation (C4) has to be limited when $z \rightarrow -\infty$. Hence it follows that $b_z = C_3 \exp(\lambda_g z)$, where $\text{Re } \lambda_g \geq 0$ and C_3 denotes the undetermined constant.

The solution of equation $\nabla^2 \mathbf{B} = 0$ for the atmosphere has to be limited when $z \rightarrow \infty$. The vertical component of this solution is given by $b_z = C_4 \exp(-kz)$, where C_4 is one more undetermined constant. The component b_x is conveniently derived from Maxwell equation $\nabla \cdot \mathbf{B} = 0$ which reduces to $b_z' + ikb_x = 0$. Since the sea floor curvature and the curvature of free-surface fluid caused by waves are disregarded, we require that b_z and b_x must be continuous at the boundaries $z = 0$ and $z = -h$. After finding the constants $C_1 - C_4$ from these conditions, one can bring the solution for the magnetic perturbations to the form given by

$$B_{Az} = \frac{kV_0 q_1 \exp(-kz)}{\omega w_1 \sinh(kH)}, \quad B_{Ax} = -iB_{Az}, \quad B_{Ay} = 0, \quad (\text{C5})$$

while for the fluid layer ($-H < z < 0$) the solution of the problem can be written as

$$B_{fz} = \frac{kV_m}{\omega \sinh(kH)} \left\{ \frac{[q_2 \exp(\lambda z) + q_3 \exp(-\lambda z)]}{2w_1} + q_4 \right\}, \quad B_{fy} = 0, \quad (\text{C6})$$

$$B_{fx} = \frac{iV_m}{\omega \sinh(kH)} \left\{ \frac{\lambda [q_2 \exp(\lambda z) - q_3 \exp(-\lambda z)]}{2w_1} + kq_5 \right\}.$$

Here we made use of the following abbreviations

$$\begin{aligned} q_1 &= w_2 k \{B_{0x} \cosh(kH) - iB_{0z} \sinh(kH)\} - w_3 \lambda \{B_{0x} \sinh(kH) - iB_{0z} \cosh(kH)\}, \\ q_2 &= (k - \lambda) (iB_{0z} \lambda_g + kB_{0z}) + k(\lambda + \lambda_g) (B_{0x} - iB_{0z}) \exp\{(\lambda + k)H\}, \\ q_3 &= k(\lambda - \lambda_g) (B_{0x} - iB_{0z}) \exp\{(k - \lambda)H\} - (\lambda + k) (iB_{0z} \lambda_g + kB_{0z}), \\ q_4 &= iB_{0z} \cosh\{k(z+H)\} - B_{0x} \sinh\{k(z+H)\}, \\ q_5 &= iB_{0z} \sinh\{k(z+H)\} - B_{0x} \cosh\{k(z+H)\}, \\ w_1 &= \lambda(\lambda_g + k) \cosh(\lambda H) + (\lambda^2 + \lambda_g k) \sinh(\lambda H), \\ w_2 &= \lambda \cosh(\lambda H) + \lambda_g \sinh(\lambda H), \quad w_3 = \lambda_g \cosh(\lambda H) + \lambda \sinh(\lambda H). \end{aligned} \quad (\text{C7})$$

For the fluid layer ($-h < z < 0$) the electric field components e_x and e_z are conveniently derived from Maxwell Equation (3). Taking into account that the projections of the vector $\nabla \times \mathbf{B}$ onto axes x and z are equal to zero, we obtain that $e_x = V_z B_{0y}$ and $e_z = -V_x B_{0y}$.

For the atmosphere ($z > 0$) we use the equation $\nabla^2 \mathbf{E} = \mathbf{0}$ whence it follows that

$$\frac{d^2 e_{x,z}}{dz^2} - k^2 e_{x,z} = 0. \quad (\text{C8})$$

This equation admits the solution $e_x = C_0 \exp(-kz)$ subjected to the condition that e_x has to be limited when z tends to infinity. Taking into account that tangential component of electric field must be continuous at $z = 0$, we get $e_x = V_m B_{0y} \exp(-kz)$. The vertical component of the electric field in the atmosphere can be derived from the equation $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$, which is reduced to $e'_x - ike_z = 0$. Substituting the derived above solution for e_x into this equation, we obtain that in the atmosphere $e_z = ie_x = iV_m B_{0y} \exp(-kz)$.

REFERENCES

1. Weaver, J. T., "Magnetic variations associated with ocean waves and swells," *J. Geophys. Res.*, Vol. 70, 1921–1929, 1965.
2. Sanford, T. B., "Motionally induced electric and magnetic fields in the sea," *J. Geophys. Res.*, Vol. 76, 3476–3492, 1971.
3. Surkov, V. and M. Hayakawa, *Ultra and Extremely Low Frequency Electromagnetic Fields*, 486, Springer Geophysics Series, XVI, Springer, 2014.
4. Surkov, V. V., "Front structure of the Alfvén wave radiated into the magnetosphere due to excitation of the ionospheric E layer," *J. Geophys. Res., Space Physics*, Vol. A101, 15403–15409, 1996.
5. Surkov, V. V., O. A. Pokhotelov, M. Parrot, E. N. Fedorov, and M. Hayakawa, "Excitation of the ionospheric resonance cavity by neutral winds at middle latitudes," *Ann. Geophys.*, Vol. 22, 2877–2889, 2004.
6. Landau, L. D. and E. M. Lifshitz, "Fluid mechanics," Vol. 6 of *A Course of Theoretical Physics*, 2nd Edition, 539, Pergamon Press, revised, 1987.
7. Havelock, T. H., "The wave pattern of a doublet in a stream," *Proc. Roy. Soc. Lond. (A)*, Vol. 121, 515–523, 1928.
8. Arzhannikov, A. V. and I. A. Kotelnikov, "Excitation of ship waves by a submerged object: new solution to the classical problem," *Phys. Rev. E*, Vol. 94, 2016, 023103, doi:10.1103/PhysRevE.94.023103.
9. Madurasinghe, D., "Induced electromagnetic fields associated with large ship wakes," *Wave Motion*, Vol. 20, 283–292, 1994.
10. Zhu, X.-J. and M.-Y. Xia, "Magnetic field induced by wake of moving body in wind waves," *Progress In Electromagnetics Research*, Vol. 149, 109–118, 2014.
11. Madurasinghe, D. and E. O. Tuck, "The induced electromagnetic field associated with submerged moving bodies in an unstratified conducting fluid," *IEEE Journal on Oceanic Engineering*, Vol. 19, 193–199, 1994.
12. Ya'akobi, O., G. Zilman, and T. Miloh, "The electromagnetic field induced by a submerged body moving in stratified sea," *Proc. of International Workshop on Water Waves and Floating Bodies (IWWWFB20)*, Longyearbyen, Norway, 2005.
13. Chang Y., F. Zhao, J. Zhang, F.-W. Hong, P. Li, and J. Yun, "Numerical simulation of internal waves excited by a submarine moving in the two-layer stratified fluid," *Proc. of the Conference of Global Chinese Scholar on Hydrodynamics*, Shanghai, China, *J. Hydrodynamics*, Ser. B, Vol. 18, 330–336, 2006.
14. Ya'akobi, V., G. Zilman, and T. Miloh, "Detection of the electromagnetic field induced by the wake of a ship moving in a moderate sea state of finite depth," *J. Eng. Math.*, Vol. 70, 17–27, 2011, doi:10.1007/s10665-010-9410-z.
15. Surkov, V. V., V. M. Sorokin, and A. K. Yashchenko, "Geomagnetic perturbations caused by a ball moving in a conducting fluid," *Radiophysics and Quantum Electronics*, Vol. 60, 617–626, 2017.