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The Relative Umbral Area in Spot Groups as an Index of Cyclic Variation of Solar Activity

N.G. Bludova · V.N. Obridko · O.G. Badalyan

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Abstract The Greenwich series of data was used to study the ratio [q] of the total umbra area to the total area of the sunspot group (for brevity "relative umbral area") for the period 1874 – 1976. It was revealed that the annual mean value of q varied in time from 0.15 to 0.28 and reached its maximum in the early 1930s. The dependence of q on the sunspot group area [S] was considered to show that the smallest groups, of area less than 100 m.v.h. (millionths of the visible hemisphere), contributed most significantly to the temporal variation of q. In contrast to the earlier results, the dependence obtained proved to be rather complicated. The coefficients of the linear expansion $q(S)$ are themselves dependent on the sunspot-group area and time [t]; *i.e.* the relation of q to both S and t is nonlinear. Only in sunspot groups with a large area does dependence disappear, and q becomes constant, equal to 0.18. This is the value given in textbooks. The relations obtained show that the relive umbral area and the relative number of small groups are important parameters of the secular variation of solar activity. In particular, they may account for variations in the mean magnetic field in active regions, the complexity of a group according to the magnetic classification, the flare activity of a sunspot group, and its geophysical impact. It is conjectured that the parameter q describes the time-varying relative contribution from the interior and subsurface dynamo mechanisms.

Keywords Sunspots · Sunspot group structure · Relative umbral area · Cyclic variation of activity

1. Introduction

When analyzing the cyclic variations of solar activity, researchers usually pay attention to variations in the number or intensity of certain objects. In fact, it is assumed that sunspots, faculae, and quiet photosphere remain the same in different phases of the activity cycle.

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There are very few publications that deal with the analysis of structural variations of these objects.

At first sight, the question of the relative area of the sunspot umbra seems to be a special problem of minor importance. However, the fact that the umbral area changes regularly with time and phase of the solar-activity cycle and depends on the rotation rate and intensity of processes in sunspots made us change our attitude to this phenomenon. It cannot be accounted for by the mechanisms of solar activity available, which treat the sunspot as a whole.

A stable sunspot consists of two basic components: umbra and penumbra. The mechanism that generates the cool magnetic umbra must also increase the penumbra with its typical properties (Evershed and Wilson effects). It is well known that the sunspots *per se* are manifestations of concentrated strong (a few kilogauss) magnetic fields. The concentration of the magnetic field in a sunspot is a complicated process not yet fully explained (Mogilevsky *et al.*, 1968; Parker, 1974a, 1974b, 1981, 1989).

In this work, we will consider variations in the sunspot structure, *i.e.* the temporal variation of the relative area of the sunspot umbra [q] (or more exactly, the ratio of the total area of the umbra to the total area of all sunspots in a group):

$$q = S_{\mathrm{U}}/S, \tag{1}$$

where S_{U} and S are, respectively, the total areas of the umbrae and the whole sunspot group. Note that by the umbra area is meant the total area of the umbrae of all sunspots in a group rather than the area of the umbra of an isolated spot. For brevity, we use the term "relative umbral area", which means the ratio of the umbral area to the total area of the group.

When analyzing isolated spots, one often uses also the ratio of the sunspot diameter to that of the umbra [$F = D_{\mathrm{P}}/D_{\mathrm{U}}$, here D_{P} and D_{U} are, respectively, the penumbra and umbra diameters]. Then, our value q is related to F as follows: $q = 1/F^2$. In the subsequent discussion, we will use the relative area of the sunspot umbra determined above in Equation (1). When citing original articles, we will give the value of q that we obtained from the results of the work referred to.

The dependence of the relative area of the umbra on other sunspot characteristics and on time was considered by various authors (*e.g.* Waldmeier, 1941; Dezsö and Gerlei, 1964; Vitinsky, 1982; Antalová, 1971, 1991 and references therein). Most of these studies are based on the Greenwich sunspot areas (solarscience.msfc.nasa.gov/greenwch.shtml). Here, we are using the data for the period 1874 – 1976, which include both the areas of sunspot groups and the total areas of the umbrae of these groups.

Nicholson (1933) and Waldmeier (1939) showed that, in isolated, circular spots, the diameter of the penumbra (or the entire spot, which is the same) relates to the diameter of the umbra as $F = D_{\mathrm{P}}/D_{\mathrm{U}} = 2.4$. This yields $q = 0.174$. The cycle dependence of $D_{\mathrm{P}}/D_{\mathrm{U}}$ was not considered in these articles.

Jensen, Nordø, and Ringnes (1955) were the first to study the cycle variation of the total sunspot area to umbra area ratio, *i.e.* the inverse of q. Using the Greenwich sunspot areas, they found that, for regular spots:

i) $1/q$ was larger in the epochs of 11-year maximum than in the epochs of minimum;
ii) $1/q$ increased with the increase of the sunspot area [S].

These conclusions were corroborated by Tandberg-Hanssen (1955) and hold true for bipolar and complex groups.

Bray and Loughead (1964) reviewed these studies and pointed to a significant inconsistency and unreliability of the results obtained. They considered reliable only two conclusions:

i) the mean relative umbral area $[q]$ is equal to 0.17, although its individual values may differ significantly from spot to spot; and
ii) the mean value of q is somewhat lower at the maximum of the cycle than at the minimum.

In their review, Bray and Loughead ignored completely the conclusion of Waldmeier (1939) that q increases with the increase of the sunspot area, which contradicts the results of Jensen, Nordø, and Ringnes (1955). Moreover, they thought that a further statistical analysis of this relation unreasonable, because observations without special facilities for reducing the stray light were too much subject to uncertainty.

In succeeding years, statistical studies of the relative umbral area as a function of time and sunspot parameters were very scarce. A series of studies was carried out by Antalová (1971, 1991). To describe this ratio, she used an alternative index (let us denote it q_{a}), which was calculated from the formula

$$q_{\mathrm{a}} = W/U - 1, \tag{2}$$

where U is the annual mean area of the sunspot umbra, and W is the annual mean total area of the corresponding spot. This means that Antalová (1971) calculated the ratio between the areas of the penumbra *per se* and the umbra. Hence, our index q is related to q_{a} (to the accuracy of averaging) by the expression

$$q = 1/(q_{\mathrm{a}} + 1). \tag{3}$$

The results obtained by Antalová (1971) are as follows:

i) The q_{a} index differs significantly in the epochs of minimum of different 11-year cycles; *e.g.* in 1901 $q_{\mathrm{a}} = 2.2$ and in 1885 $q_{\mathrm{a}} = 7.1$. This means that q was equal to 0.31 in 1901 and 0.12 in 1885.
ii) The value of q_{a} changes from cycle to cycle. Antalová (1971) calculated the mean values of W and q_{a} over nine-year intervals in each cycle. It turned out that q_{a} was the largest (and q was, correspondingly, the smallest) in Cycle 14 when the mean area W was the smallest of all cycles considered.
iii) Throughout the time interval 1874 – 1959, q_{a} was larger at the maxima than at the minima of the activity cycles.

These results agree with the conclusions drawn by Jensen, Nordø, and Ringnes (1955) and Tandberg-Hanssen (1955) with allowance for different definitions of the magnitude under examination. Besides, the umbral and penumbral areas depend differently on the phase of the 11-year cycle. Antalová (1971) investigated in detail the behavior of q_{a} at the maximum of the activity cycle. She found that in the periods of the secondary maxima of Cycles 13 – 19, q_{a} decreased compared to the primary ones. In all sunspots with an area more than 500 m.v.h. (millionths of the visible hemisphere), the relative area of the umbra at the secondary maximum was larger than in the primary one.

As shown by Antalová (1971), q_{a} for the large spots is larger (*i.e.* q is smaller) than for the small ones. In that work, Antalová used data from the Greenwich Catalog, and all of her results refer to sunspot groups. Therefore, here, the same as above, by q is to be meant the ratio of the total area of all umbrae in a sunspot group to the total area of the group.

Later, Antalová (1991) turned to the study of individual stable spots. She selected 74 symmetric spots for the period 1968 – 1976 from the Greenwich Photoheliographic Results and 15 spots from Debrecen observations. In the first group, the maximum values of the longitudinal magnetic field had been measured at the Crimean Astrophysical Observatory and in the second one at Irkutsk (113 and 26 measurements were available, respectively).

As a result of her study, Antalová found an inverse relationship between the longitudinal component of the sunspot magnetic field [B_L] and the ratio of the penumbral diameter to that of the umbra [F]. The parameter $F = D_P/D_U$ was obtained from observations using the relation $F = (S_P/S_U)^{1/2}$. The data presented by Antalová suggest that q increases slightly from 0.17 to 0.19 as the magnetic field changes from 200 mT to 300 mT. The figures in the article demonstrate a significant scatter. For example, the values of q corresponding to the field of 260 mT range from 0.15 to 0.23. This is quite understandable, since the magnetic measurements depend strongly on the seeing conditions on every particular day, the error reaching 50 – 70 mT. Furthermore, the use of the maximum field value in itself can result in a seeming correlation with the umbra area, because the point of maximum field is easily found in a large, usually inhomogeneous spot. In the 100 – 200 m.v.h. spots with the umbra of 20 – 40 m.v.h. and the field intensity of about 200 mT, the dependence between the umbral area and the magnetic field virtually disappears.

Hathaway, Wilson, and Campbell (2007) analyzed the behavior of the daily values of q using the Greenwich database. The mean value of q was 0.2 for the entire period under examination. With the increase of the mean sunspot area, this value was decreasing. According to these authors, the cycle dependence of q was weak, and the dependence on latitude was absent. An unexpected result was that q for sunspot groups of small area began to increase dramatically from 1910, became half as large again by 1930 and then returned to its "normal" value by 1950. However, the authors do not specify in their brief article what they mean by the "normal" value, neither do they consider how the ratio between the umbral and penumbral areas is related to the total sunspot area.

Thus, the studies available reveal an intricate pattern of temporal and cycle variations in the sunspot internal structure, about which there is no full agreement between different authors.

We started working on the problem in 2006 (see Bludova and Obridko, 2007). We studied the behavior of the ratio q of the umbral area to the total sunspot area over the entire period covered by the Greenwich Catalogue (1874 – 1976). In Section 2, we demonstrate the temporal dependence of q and corroborate its dramatic increase in the 1930s. We make a conjecture that the latter was partly due to the increased number of spots of small area. This hypothesis is discussed in detail below, in Sections 2 and 5. Then, we analyze the dependence of the relative area of the umbra on the sunspot group's total area (Section 3). The linear approximation of this relation is considered in Section 4 where we analyze the dependence of the approximation coefficients on time and on the total group area. The discussion in Section 5 is devoted to the relation of our results to the secular variation of sunspot characteristics and, probably, to the solar dynamo mechanisms.

2. The Relative Area of the Sunspot Umbra as a Function of Time

The ratio [q] of the total umbral area to the total area of the group was obtained using the daily data of the Royal Greenwich Observatory. The annual mean value of q was calculated for each year from 1874 to 1976. Note that we always mean a sunspot group and not an individual spot.

Note that both our results and those reported by other authors depend on the averaging procedure. For example, Antalová (1971) found the annual mean areas of the umbra and penumbra and then calculated their ratio. This procedure increases the contribution from the relatively rare large sunspots for which q is relatively low, amounting on average to 0.18. Therefore, according to Antalová the values of q range from 0.15 to 0.18. We, on the

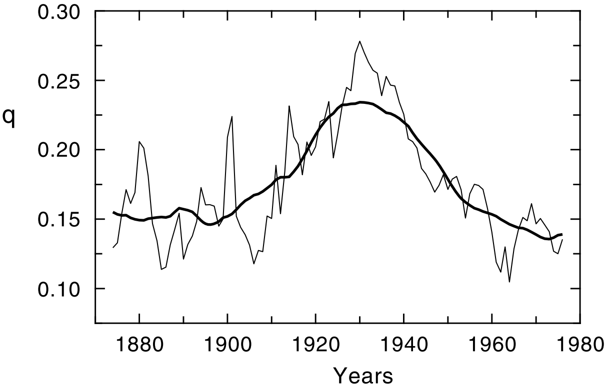


Figure 1 The ratio of the umbral area to the total area of the sunspot group (thin curve) and its moving average obtained with a sampling window of 25 years (thick curve).

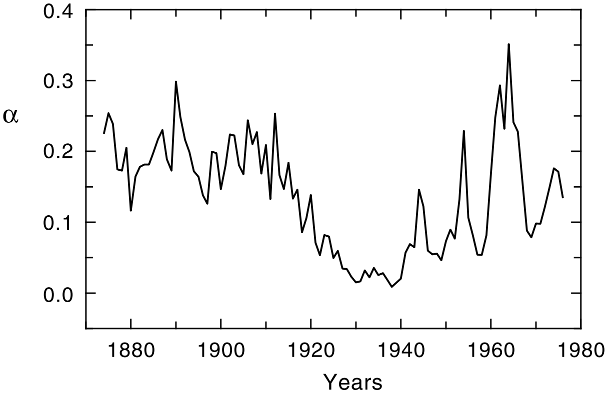


Figure 2 The number of sunspot groups without umbra relative to the total sunspot group number.

contrary, found q separately for each spot group in a given year and then calculated the annual mean. This procedure increases the contribution from the most abundant spot groups (including small ones); therefore, the variation range of q in our work as well as in the work by Hathaway, Wilson, and Campbell (2007) is much larger (0.15 – 0.29). Thus, when comparing the results of different authors, one should have in mind the averaging procedure applied, because it may lead to overestimation of the contribution of sunspot groups of different areas depending on their abundance.

Figure 1 illustrates the temporal variation of the relative area of the umbra [q]. One can see that at the end of the 19th century, q was equal, on average, to 0.15. At the beginning of the twentieth century, q started to increase and reached its maximum of 0.28 in the early 1930s. After that, it began to decrease and had dropped to 0.13 by the 1970s. This is, probably, the level that Hathaway, Wilson, and Campbell (2007) called "normal".

Observations have also revealed nonstandard objects – groups of sunspots without umbra. Figure 2 shows the relative number of sunspots without recorded umbra, [α]. One can see that this number changed significantly with time and was virtually zero during the 1930s and 1940s. This means that sunspots without umbra were scarce in the years when the annual mean relative area of the umbra q was maximum.

Note that the behavior of q plotted in Figure 1 does not change if we eliminate the spots without umbra. Independent of whether or not the sunspot groups without umbra are taken into account, the position and height of the main maximum will remain virtually the same, equal to 0.278 or 0.282, respectively. This is not surprising, since the groups without umbra

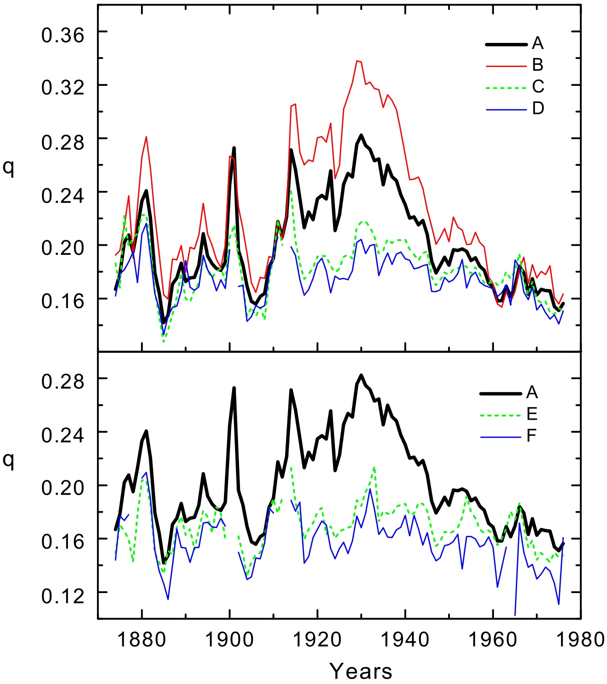


Figure 3 Upper panel, from top down: q for the spot groups of area no more than 100 m.v.h. (upper, red curve B), for all groups (thick-black curve A), for the groups of area 100 – 200 m.v.h. (dashed-green curve C), and for the groups of area 200 – 300 m.v.h. (lower blue curve D). Lower panel, from top down: q for all groups (thick-black curve A), for the groups of area 300 – 600 m.v.h. (dashed-green curve E), and for the curves more than 600 m.v.h. (lower blue curve F).

were very scarce in the period from 1927 to 1940. In other periods, the unsmoothed values of q change slightly, but the general trend of the curve in the figure remains the same. As a matter of fact, the groups without umbra are specific features that differ drastically in their properties from the spots with umbra. They are, essentially, "patches" of the penumbral matter with mainly tangential magnetic field and a short lifetime. Taking them into account in a general statistical study may distort the results. Therefore, the sunspots in which the umbra was not recorded were omitted from our further consideration.

3. The Dependence of the Relative Area of the Umbra on the Sunspot Group's Area

Figure 3 illustrates the temporal variation $[q(t)]$ for sunspot groups of different area. The black (thick) curves and symbol A on both panels show q for all sunspot groups with umbra (*i.e.* the groups without umbra are eliminated). The red (thin) curve B at the top of the upper panel illustrates q for the groups of area no more than 100 m.v.h. Thus, q for the smallest groups proves to be, on the average, larger than for all sunspot groups taken together. It was already noted above that in small spots, the area of the umbra depends only weakly on the magnetic field (Antalová, 1991). On the other hand, the groups consisting of such spots have a rudimentary penumbra and, therefore, the fraction of the umbra $[q]$ increases. This question is considered in more detail in Section 5.

The other two curves on the upper panel – the green (dashed) curve C and blue (lower thin) curve D – illustrate q for the groups of area from 100 to 200 m.v.h. and from 200 to 300 m.v.h. Curve C lacks the point for 1913, and the lowest curve D lacks the points for 1901, 1912, and 1913, *i.e.* for the years when no spots of the corresponding size were observed.

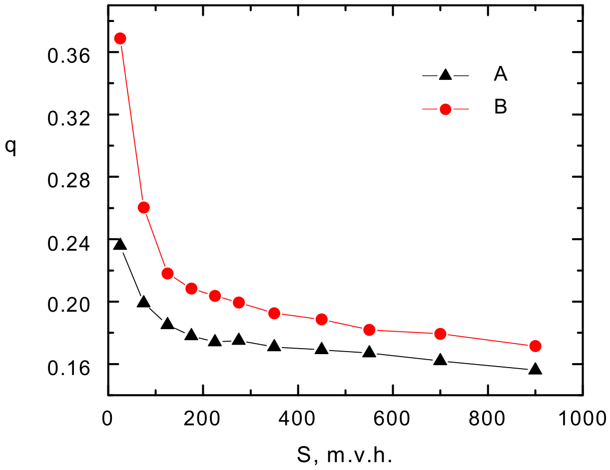


Figure 4 The mean [q] for sunspot groups of all sizes over all years under discussion (A: black curve with triangles) and, separately, for 1929 (B: red curve with circles).

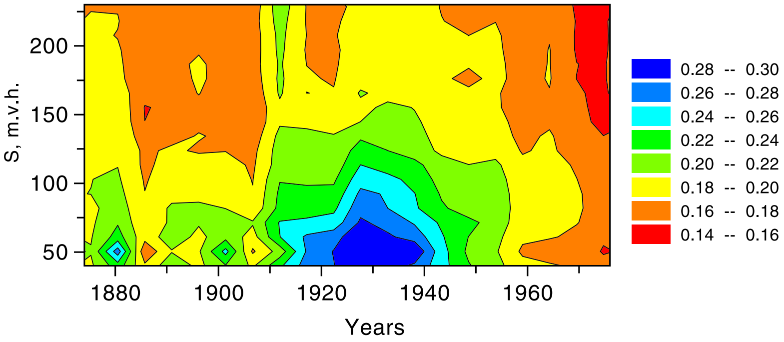


Figure 5 General distribution of q as a function of time and sunspot group area [S].

Thus, the temporal dependence of q becomes weaker as the mean size of a sunspot group increases. The mean value of q decreases gradually. The value for each year is the lower the larger is the mean sunspot group area in the given year. This decrease is also observed in the case of very large groups. In Figure 3 (lower panel) curve A for all sunspot groups is juxtaposed with the similar curves for the groups of area 300 – 600 m.v.h. (dashed-green curve E) and more than 600 m.v.h. (blue, lower thin, curve F). One can see that the decrease of q continues every year. Here, too, there are years when groups of the corresponding area were absent. These are 1900, 1901, and 1913 for the 300 – 600 m.v.h. groups and 1878, 1879, 1900, 1901, 1911, 1912, 1913, and 1964 for the groups of area more than 600 m.v.h.

Figure 4 shows the mean value of q for the entire period under consideration (curve A) and, as an example, for 1929 (curve B). One can see that q decreases with the increasing mean area of the sunspot group. The same is true for any other year.

Figure 5 illustrates the conclusions drawn above. It represents the general distribution of q in the time–area reference frame. Different colors denote eight ranges of q. Red corresponds to the smallest values, and blue to the largest ones. It is readily seen that the parameter q increased in the 1930s. The increase can be revealed for the sunspot groups of areas up to 200 m.v.h., but it is best pronounced for the smallest groups. The figure shows also that

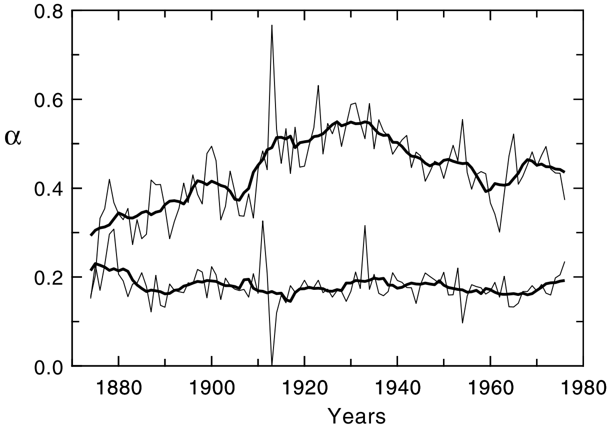


Figure 6 The relative number of sunspot groups of area less than 100 m.v.h. (upper curves) and $100 < S < 200$ m.v.h. (lower curves). The thin curves illustrate the annual mean values and the thick ones, the values smoothed by a nine-year window.

the mean value of q was decreasing with the increase of the group area throughout the time interval under consideration.

Figure 6 represents the temporal dependence of α – the ratio of small sunspot groups with umbra of areas less than 100 m.v.h. (upper curves) and 100 – 200 m.v.h. (lower curves) to the total number of groups (including the spots without umbra) in each year. The thin curves show the annual mean values and the thick ones, the values smoothed by a nine-year window. As seen in Figure 6, the fraction of sunspot groups with small area was quite substantial (more than 50 %) throughout the period under examination. It reached its maximum in the early 1930s, *i.e.* at the minimum and rise of a relatively low Cycle 17 (the Wolf number at the maximum equaled 119.2). Then, α decreased gradually by the early 1950s reaching its minimum at the maximum of the record-high Cycle 19. At the same time, the fraction of the groups of area $100 < S < 200$ m.v.h. did not, in fact, depend on time. On the whole, the fraction of sunspot groups of area less than 200 m.v.h. ranged from 50 % to 80 % of the total number. Correspondingly, the fraction of the groups of area $S > 200$ m.v.h. dropped to the minimum in the period 1920 – 1940. Somewhat unexpected is the absence of an 11-year periodicity for these ranges, although a clear secular dependence does exist.

The fact that the mean value of q varies with time might be explained by the change in the relative number of small sunspot groups. This conclusion is suggested by Figures 3 and 6. It should be noted, however, that a simple increase of the number of small groups cannot account for all effects observed in the 1930s. For example, one can see from Figures 4 and 5 that in 1929 the relative umbral area [q] was higher than the mean value for the groups of all sizes. This suggests that the maximum in Figure 1 is determined not only by the increased number of small sunspot groups, but also by a certain additional, yet unknown, factor (see Section 5 for a more detailed reasoning). It is readily seen that some of the results described above differ from those reported by Waldmeier (1939). The causes of such a discrepancy will be discussed below, in Section 5.

4. Linear Approximation of the Dependence of *q* on the Sunspot Group's Area

The dependence of the relative area of the umbra [q] on the total sunspot group area [S] can be expressed by the following equation, which yields a linear expansion:

$$q = a_0 + a_1 S. \tag{4}$$

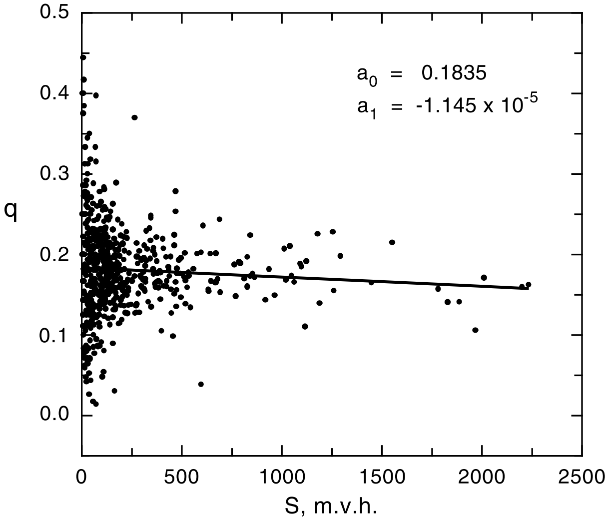


Figure 7 The relation between q and the total sunspot group area [S] for 1898. The values of q for sunspot groups observed in 1898 are shown with dots; the straight line represents the regression equation. The corresponding values of a_0 and a_1 (Equation (4)) are specified.

Such a relation for the sunspot groups recorded in 1898 is shown by way of example in Figure 7. When plotting the relation, we omitted the spots without an umbra (in 1898, they constituted 20 % of the total number of sunspot groups). The coefficients in Equation (4) for 1898 are $a_0 = 0.1835$ and $a_1 = -1.145 \times 10^{-5}$. Figure 7 shows that the relative umbral area decreases with the increase of the area of sunspot groups as was pointed out above. Of course, taking into consideration sunspot groups without umbra might distort the diagram owing to the points at the lower-left corner. However, as mentioned above, such groups form a specific class of solar-activity features and including them in the general statistics is inappropriate.

Let us consider the temporal behavior of the coefficients a_0 and a_1. As seen in Figure 7, the linear approximation describes the real dependence of q on S more or less satisfactorily in the case of relatively large areas: $S > 300$ m.v.h. At $S \leq 300$ m.v.h., the dependence does not disappear, but the spread is so strong that the reliability of the linear approximation becomes dubious.

The upper panels in Figure 8 represent the temporal dependence of the coefficients a_0 and a_1 for all sunspot groups. One can see that the behavior of a_0 is very similar to that of q in Figure 1. On the other hand, a_1 apparently demonstrates a negative correlation with a_0.

In order to find which spots influence the temporal dependence of q most significantly, we have considered separately the sunspot groups of area less and more than 300 m.v.h. For these two populations, we determined the corresponding coefficients a_0 and a_1. They are shown in Figure 8 – the middle and lower panels for the small and large sunspot groups, respectively. One can see that the curve on the left-middle panel, *i.e.* the coefficient a_0 for the spot groups of area less than 300 m.v.h., demonstrates the same temporal dependence as q does in Figure 1: the main particularity in the behavior of both q and a_0 for small groups is a pronounced maximum in the 1930s. Figure 8 illustrates the suggestion made above that the principal role in the temporal variation of the parameter q belongs to small sunspot groups.

In order to analyze the dependence $q(t)$ for the groups of area less than 300 m.v.h. in more detail, they were divided into three groups: $S \leq 100$ m.v.h., $100 < S \leq 200$ m.v.h., and $200 < S \leq 300$ m.v.h. The result is illustrated in Figure 9.

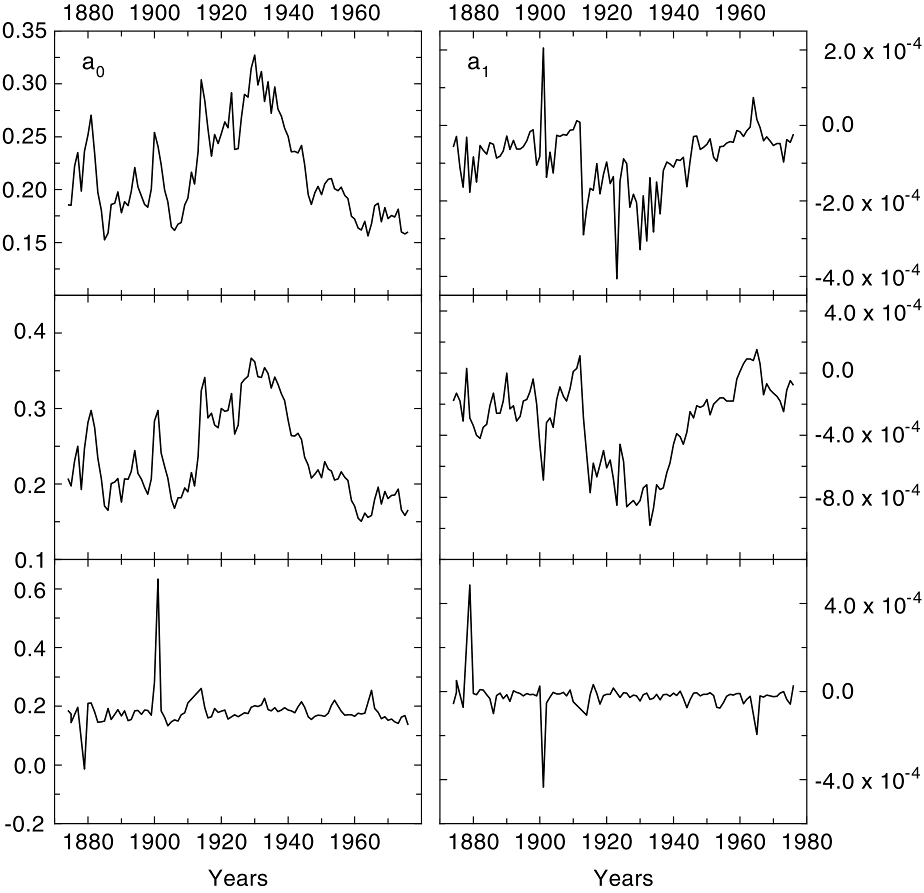


Figure 8 Coefficients a_0 (left) and a_1 (right) as derived from Equation (4) for all sunspot groups (top), for the groups of area less than 300 m.v.h. (middle), and for the groups of area more than 300 m.v.h. (bottom).

Figures 8 and 9 show that:

i) The curves a_0 and a_1 for the smallest groups (upper panels in Figure 9) and for all spots with umbra (upper panels in Figure 8) are very similar. The maxima of the a_0 curves occur in the 1930s when the a_1 curves display minimum values.
ii) In Figure 9, the middle panels illustrate the transition from small to large groups, while the curves on the lower panels have little in common with the upper ones and resemble more the curves for the > 300 m.v.h. spots represented on the lower panels in Figure 8.
iii) As the area of sunspot groups increases, the parameter a_0 gradually approaches a constant value of about 0.18. Simultaneously, the parameter a_1 approaches zero.

Table 1 presents the mean values of a_0 and a_1 for the populations of sunspot groups considered above. The first column shows the sunspot-group area, the second column is the mean value of a_0, and the third one the mean value of a_1. The fourth column is the number of years $[N]$ when the spots of the given area were recorded. The table shows that for the largest groups of area up to 300 m.v.h., the standard error of the mean exceeds the a_1 value *per se*, *i.e.* this parameter, on the average, equals zero.

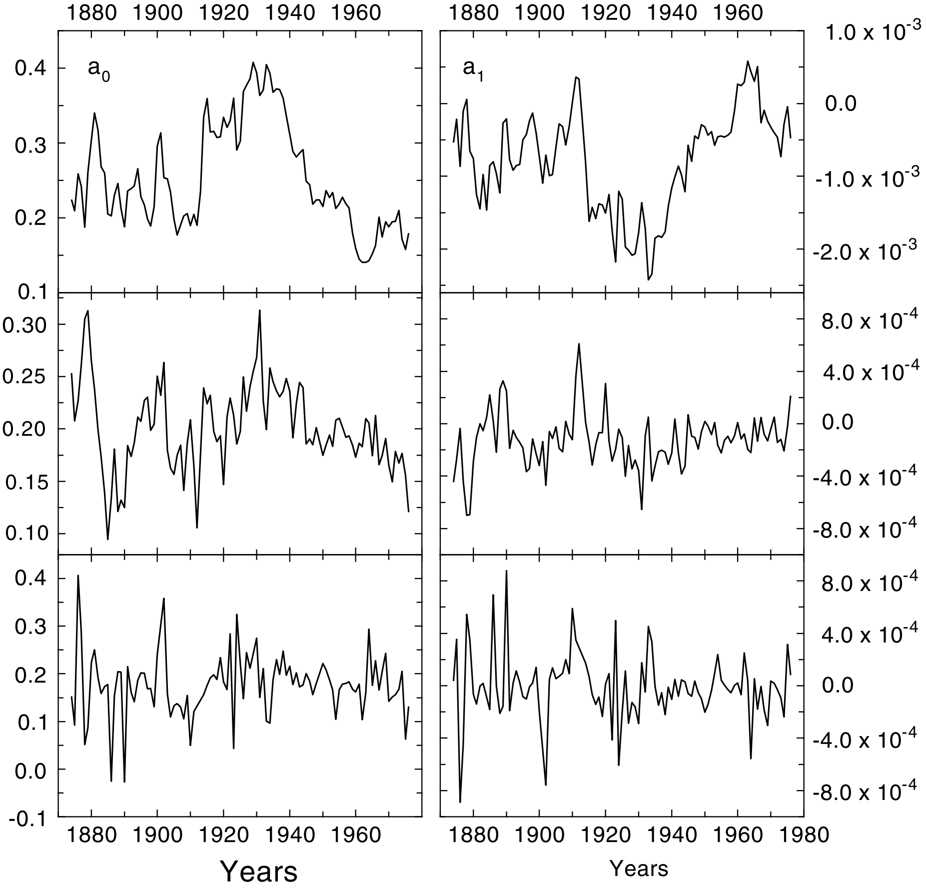


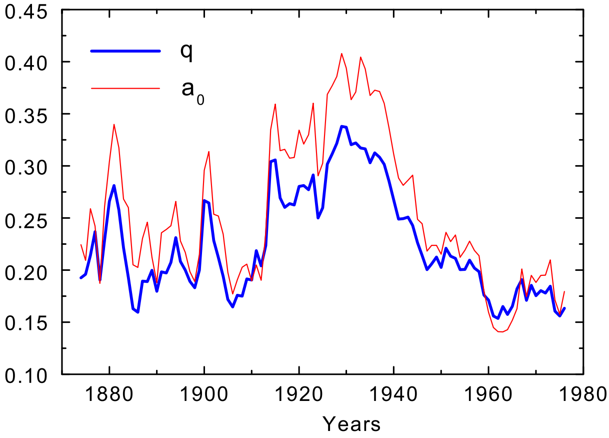
Figure 9 Coefficients a_0 (left) and a_1 (right) for the sunspot groups of area $S \leq 100$ m.v.h. (top), $100 < S \leq 200$ m.v.h. (middle), and $200 < S \leq 300$ m.v.h. (bottom).

Table 1 The mean values of a_0 and a_1 for different sunspot group areas.

S	a_0	a_1	N
< 100	0.2543 ± 0.0070	-0.000778 ± 0.000068	103
100 – 200	0.2008 ± 0.0041	-0.000130 ± 0.000020	102
200 – 300	0.1770 ± 0.0066	-0.000011 ± 0.000026	100

Comparing the upper-left panel in Figure 9 with Figure 1, which illustrates the dependence $q(t)$ for all sunspot groups without dividing them into small and large ones, we can see that both curves are very similar. In Figure 10, we have juxtaposed the curves a_0 and q for the smallest groups of area less than 100 m.v.h. The comparison reveals a close similarity between these curves, too; *i.e.* the ratio of the total area of the umbra to that of the entire group for the smallest groups behaves in the same way as the ratio for all sunspot groups, including the spots without umbra. The comparison confirms also that numerous small groups contribute to the behavior of $q(t)$ most significantly.

Figure 10 Comparison of the a_0 and q curves for the smallest sunspot groups.



The correlation between q and a_0 for the three populations of sunspot groups considered above is as follows: for the sunspot groups of area $S \leq 100$ m.v.h., the correlation coefficient r is 0.97; for $100 < S \leq 200$ m.v.h., $r = 0.51$; and for $200 < S \leq 300$ m.v.h., $r = 0.24$.

We can draw a conclusion that the relation between q and S is nonlinear, and its characteristics depend on S. For the very small sunspot groups q is large, reaching the maximum value of about 0.35. As the group area increases, the relative area of the umbra decreases. At S from 200 to 300 m.v.h., it takes on a certain universal value, which remains constant. On the other hand, q displays a distinct temporal dependence, but does not show any relation with the 11-year cycle and its phases.

A high anticorrelation between a_0 and a_1 is the result of plotting the linear regression by the least-squares method. Any function can be, obviously, approximated by a linear function over a short variation interval of the argument (*e.g.* by using a Taylor series). In this case, the regression coefficient a_1 of the relation obtained for the decreasing function (see Equation (4) and Figure 7) is, naturally, negative. Since an exponential and a linear functions on a short interval differ little, the regression coefficient in the linear expansion is close to the exponential index, which results in a significant correlation between the coefficients a_0 and a_1.

5. Discussion

We have investigated the relationship between the total areas of the umbra and the sunspot group as a whole; for brevity, we use the term "relative umbral area". The study was based on the Greenwich data for 1874 – 1976, *i.e.* the period for which the umbral areas are available in the Greenwich Catalog. The parameter q was determined separately for each group and, then, its annual mean was calculated.

It turned out that the annual mean ratio $[q]$ of the umbral area S_U to the total group area S, *i.e.* $q = S_U/S$, was changing significantly over the 103-year period under consideration, ranging from 0.15 up to 0.28 (Figure 1). This is a long-term variation not connected with the 11-year cycle.

Most noticeable in the behavior of $q(t)$ are the characteristic features observed in the 1930s. At that time:

i) the variation of q demonstrates a pronounced maximum, which was noticed also by Hathaway, Wilson, and Campbell (2007);

ii) the specific structural features – sunspots without umbra – actually disappeared;
iii) the fraction of relatively small (up to 100 m.v.h.) sunspot groups increased (Figure 6).

These is evidence of changes that occurred in the sunspot formation activity of the Sun in the 1930s.

On the whole, it is of fundamental importance that the proportion of sunspot groups of different sizes reflects a certain long-term variation in the sunspot formation activity. It might be expected that this proportion would change over an 11-year cycle. In fact, however, we are dealing with some secular mechanism, which affects precisely the sunspot groups of small area.

Although the relative number of small sunspot groups (with recorded umbrae, see Figure 6) is always large, amounting normally to about 30 %, their percentage was increasing gradually from 1915 to 1930, reached a maximum in the early 1930s, and then decreased again down to the normal value by the late 1960s. It is important to note that, in the period under discussion, the small sunspot groups were not only more numerous, but the value of q in these groups was much higher than usual. This explanation seems to be the simplest and most natural (see Figures 3 and 6). However, as seen from Figures 4 and 5, the value of q in 1929 was higher than normal for all sunspot groups. This means that the increase in q was caused not only by the increased number of small sunspot groups but also by some other, as yet unknown, factor.

Let us recall the finding by Antalová (1991) that the area of the umbra depends on the magnetic-field intensity. Hence, the change observed in the relative number of small sunspot groups may be associated with a secular variation in the intensity of the sunspot magnetic field. Although this suggestion is quite paradoxical, recall the latest results by Penn and Livingston (2011), which showed that the intensity of the magnetic field in sunspots was decreasing and the brightness of the umbra was increasing during the past 20 years. Thus, our results described above might explain the Penn and Livingston result by a gradual decrease in the relative number of small sunspot groups. It is possible that, simultaneously, the value of q increased. Unfortunately, the sunspot umbral areas for the period considered by Penn and Livingston (2011) are missing in the Greenwich Catalog. However, the recent processing of magnetic observations of sunspot groups of different areas (Pevtsov *et al.*, 2011; Nagovitsyn, Pevtsov, and Livingston, 2012) has shown that the Penn–Livingston result can indeed be interpreted as a result of the increased fraction of small sunspot groups. On the one hand, this allows us to expect a deep minimum of solar activity within the nearest decade and, on the other, suggests the existence of two dynamo mechanisms in the Sun responsible for the formation of small and large sunspots.

The dependence of q on the group area proves to be quite complicated. It is nonlinear, and the linear-expansion coefficients, in turn, depend on the group area and time. Only for large groups, this dependence disappears: q becomes constant, equal to 0.18. This is the value given in the literature.

Our results show that the relative area of the umbra with respect to the total area of a sunspot group decreases with the increase of the latter. On the other hand, Waldmeier (1939) obtained the opposite result for a relatively small number of isolated circular spots recorded in the epochs of the cycle maximum. Waldmeier represented his data by the formula:

$$P/U = c_0 + c_1 P/P_{\odot}. \quad (5)$$

Here P is the diameter of the penumbra, U is the diameter of the umbra, and $P_{\odot}$ is the diameter of the Sun. Using the conventions from our Equation (1), we obtain

$$q = S_{\mathrm{U}}/S = (P/U)^{-2}. \quad (6)$$

Waldmeier used $c_0 = 3.2$ and $c_1 = -36.0$ in his formula. Then, we should expect q to range from 0.12 (if the group area is 100 m.v.h.) to 0.29. The latter corresponds to the spot area of 1000 m.v.h. In the same article, however, Waldmeier indicates $P/U = 2.2-2.4$ as the most frequent value, which yields $q = 0.17-0.21$. This is precisely the value (0.17) cited by Bray and Loughead (1964). Allen (1973) provides simply the ratio of the radii of the umbra and penumbra equal to 0.42, which yields $q = 0.18$. As seen from Figure 1, the annual mean value of q according to our calculations changes with time from 0.12 to 0.28, and its smoothed value changes from 0.15 to 0.23 (see also Bludova and Obridko, 2007; Hathaway, Wilson, and Campbell, 2007). Mergentaler (1986) showed that this value depends also on the Zürich class of the sunspot group. The correlation between S_{U} and S is clearly seen for the groups of class J, but is completely absent for the groups of class E.

Note a fundamental difference of our results from those reported by Waldmeier (1939). Waldmeier deliberately selected isolated circular spots. Most of them were observed in 1896, 1897, 1907, and 1917. In addition, he used a small number of sunspots observed in 1928. Thus, all data belong to the epochs of solar maximum. For the analysis, all spots were divided into two groups comprising 59 and 23 spots. Waldmeier (1939) arrived at a conclusion that the fraction of the umbra area for *individual* circular spots increases with the increasing area of the spot. We have used an alternative parameter, namely, the ratio of the *total* area of the umbra of all spots in a group to the *total* area of the group. It is not obvious that these two parameters (Waldmeier's and ours) are equivalent. Note that Figure 7 was plotted for sunspot groups observed in the same period as the individual spots studied by Waldmeier (1939). It is seen from the figure that the ratio between the total areas of the umbra and penumbra in that period decreased with the increase of the group area.

It is important to note that if the decrease of q in isolated spots revealed by Waldmeier (1939) is true, the decrease of the mean value of q in large sunspot groups revealed in our work may suggest the appearance of many small spots in those groups.

On the other hand, Brandt, Schmidt, and Steinegger (1990) showed that the ratio of the umbra to penumbra proper, in 126 sunspots at the maximum of activity in 1980, was 0.24 for small spots and 0.32 for the large ones. This yields $q = 0.19-0.24$ and agrees with the Waldmeier's conclusion of the increase of q in large sunspots. However, the method used by Brandt, Schmidt, and Steinegger (1990) to delimit the umbra and penumbra differs significantly from that applied in other works. They adopted the photometric values given by Grossmann-Doerth and Schmidt (1981), who considered the typical brightness at the umbra–penumbra and penumbra–photosphere boundaries to be equal, respectively, to 0.59 and 0.85 of the undisturbed photosphere brightness. The reliability of the boundaries determined in such a way can be questioned and are disputed by many authors who suggest alternative values of the boundary brightness (Obridko, 1985). Moreover, the method itself is dubious, because it ignores the inner bright and dark features in the umbra and penumbra and, besides, it may be strongly affected by the seeing conditions. These are the reasons why the results by Brandt, Schmidt, and Steinegger (1990) are difficult to compare with those obtained by other authors. Besides, it should be recalled that Waldmeier and Brandt and other authors analyzed individual sunspots and not sunspot groups as is done in our article.

Thus, the value of q in individual, well-developed, symmetric spots or leader spots in the group is $0.17-0.19$. As the magnetic field in such spots increases, q decreases a little, remaining within the same range (Antalová, 1991). This may imply that, with the increase of the magnetic field, the penumbra grows somewhat faster than the umbra. Note that the areas of individual spots by Antalová (1991) are significantly larger than the areas of small groups in our study.

In small individual sunspots, the field is not much weaker than in the spots in a group (no less than 150 mT), but, above all, q in such spots is significantly larger. This may be the result of observational conditions: the observer may overestimate the size of the umbra in small spots. In our case, it is not important; such a spot will be included in the catalogues with large q. Small sunspot groups (Zürich classes *A, B*) consist mainly of small spots, and the value of q in them remains large. Then, as the group grows in area (Zürich classes *E, F*), a large leader spot with a small q appears, and the total q value of the group decreases. The further evolution may take either of two courses: the group may turn into a large individual spot (class *H*), and q will remain small, or it may become a group of the more commonly encountered class *J* consisting of a few small spots, and q will increase again.

The increase in q in the early 1930s can be related to the apparent, although as yet unexplained, anticorrelation between the observed rotation rate of the Sun and the corresponding Wolf-number maximum. It turns out that the solar rotation rate was higher in the 1930s, *i.e.* in the period of low activity cycles (Hathaway and Wilson, 1990; Obridko and Shelting, 2001; Badalyan, 2011). It was in the epoch of low cycles from 1915 to 1940 that we observed a decrease in the effective rotation period of the Sun (*i.e.* an increased angular rotation rate). Moreover, it was revealed (Antalová, 1986) that the differential-rotation rate decreased in the zones where the integral sunspot area was larger.

Let us recall that Eddy, Gilman, and Trotter (1976) revealed from the old sunspot sketches by Johannes Hewelius that the rotation rate of the Sun in the equatorial zone in the period 1642 – 1644 was by 3 – 4 % higher than in the first half of the twentieth century. At the same time, the rotation rate at the latitude of 20° and higher was the same as in the first half of the twentieth century. This means that the differential character of the solar rotation was much more pronounced. However, this conclusion was questioned by Abarbanell and Wöhl (1981).

So far, all models of the generation of solar magnetic fields have tried to explain more or less successfully the intensity of processes. In fact, the majority of the dynamo models available still predict the generation of a large-scale diffuse field. Now, a number of theoretical models have appeared that consider the concentration of the magnetic field in flux tubes observed as sunspots (see, for example, Fisher *et al.*, 2000; Fisher, Chou, and McClymont, 2010; Pipin and Kosovichev, 2011; Ilonidis, Zhao, and Kosovichev, 2011). The field is concentrated in subsurface layers, so that the depth of sunspots does not exceed 10 000 km. This process may change in time and does not cease after the visible spot has appeared. Moreover, the formation of a sunspot does not cease after its emergence on the surface, but continues throughout its life time.

It is well known that a newly emerged small sunspot has but a rudimentary penumbra or none at all. Such are the overwhelming majority of sunspots (Obridko, 1985). They belong to the McIntosh class *Axx* or *Bxo* (McIntosh, 1972) and have short lifetimes not exceeding a few days. It seems to be the absence of the penumbra that makes these spots unstable. The Evershed effect in the penumbra ensures the stability of larger, long-lived sunspots (Obridko and Badalyan, 1977). As the sunspot grows, both its umbra and penumbra increase. However, the penumbra is likely to grow faster, so that the relative umbral area decreases asymptotically.

Note that γ-class sunspots were absent in 1930 – 1935, which may point to a decreased power of the dynamo in subsurface layers (Bludova, 2011).

Thus, the relative umbral area is an important parameter of evolution of the solar activity. It is likely to reflect the relative contribution from the deep and subsurface dynamo mechanisms. A gradual increase in the number of small spots may manifest the decrease in the intensity of local fields and, *vice versa*, the apparent decrease in the mean field intensity may be an instrumental effect associated with the reduced spot area.

Note that the Maunder minimum might have been a manifestation of the reduced area of sunspots. As a result, the observations with low-resolution facilities available in the 17th century (to say nothing of the naked-eye observations) gave an impression that the number of sunspots decreased. Besides, the small sunspot groups usually cause weak geophysical disturbances. Therefore the sunspot numbers determined from indirect evidence (*e.g.* aurora observations) must have been underestimated.

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