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A Comparative Analysis of the Properties of the Magnetic Fields in Leading and Trailing Sunspots

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Abstract—Pairs of leading and trailing sunspots whose umbrae are joined by magnetic-field lines have been selected based on calculations using SOLIS magnetic-field data in a potential approximation and the B_d technique of Rudenko, together with SDO data for 2010–2013. The shape of the field lines reflects to some extent the shape of the magnetic tube connecting the leading and trailing spots. The minimum angle between the field lines and the radial direction $\alpha_{\min}$, the maximum magnetic field $B_{\max}$, the length of the field line from the leading spot to the apex, where the radial component of the field is zero, L_l, and the length of the field line from the apex to its eastern base L_f are determined in the umbrae of all the selected sunspots. In ∼81% of cases, $\alpha_{\min}$ is smaller in the leading spot than in the trailing spot. For such sunspots, there is a positive correlation between these angles in the leading and trailing spots. The dependences of $\alpha_{\min}$ on the areas of the umbrae in the leading and trailing spots are different. There is a weak negative correlation between $\alpha_{\min}$ and $B_{\max}$. In other words, on average, the field lines are closer to radial in magnetic tubes forming the umbrae of both leading and trailing spots with stronger fields at the photospheric level. In ∼60–65% of cases, the section of the field adjacent to the leading spot L_l is shorter than L_f. Similar results are obtained for large single spots.

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1. INTRODUCTION

Sunspots are features of the solar atmosphere characterized by a reduced temperature and brightness at the photospheric level, and also increased magnetic-field strengths compared to other parts of the solar photosphere [1–3]. The number of spots simultaneously observed on the Sun is one of the main characteristics of solar activity, including its cyclicity [1, 4]. Sunspots are closely related to other manifestations of solar activity, such as flares in spots.

The properties of sunspot magnetic fields have been a subject of intense study from the first magnetic-field measurements in sunspots [5] up through the present. According to various studies, the magnetic fields in pores exceed 1200–1800 G, and the fields in large, well developed spots can reach 3500–4000 G. The highest magnetic fields detected in the umbra of a sunspot are about 5000–6100 G [6]. Although the magnetic-field structures in umbrae are generally fairly complex, some mean characteristics of sunspot magnetic fields have been estimated [2] (here and below, when referring to results obtained before 1985, we will in some cases refer to [2] without citing the original studies included in the monograph [2]). The magnetic field decreases appreciably with distance from the core of a sunspot umbra toward the umbra–penumbra boundary, while the angle of the field to the vertical increases from ∼0° to ∼70°–90°. In many cases, the projection of the magnetic field onto the plane of the sky is not radial; as a rule, the deviation of the perpendular field component from the radial direction at the spot center (i.e., from the spot location with the minimum brightness) increases with distance from the spot center. This testifies to some twisting of the field lines relative to the vertical direction. Fine structure of the magnetic field is observed in umbrae, such as bright points or granulation, with the field strength differing in individual elements of this structure.

It is believed that the magnetic fields in sunspot umbrae and adjacent regions of the penumbra are close to force-free [7]. Various attempts have been made to determine the variation of the magnetic field with height. It was shown in [8] that the radial component of the magnetic fields in the umbrae of small spots in the photosphere decreases with height by from 0.5 to 2.2 G/km, depending on the position

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in the spot. The electric currents and helicity of the field in a sunspot were also estimated in [8], making it possible to draw conclusions about the dependence of the electric current density and the helicity of the field on the fine structure in the spot. The electric-current density varies around several mA/m^2. The mean value for sunspot umbrae is 11 mA/m^2, while the mean value for penumbrae is 2 mA/m^2.

It was shown in [9] that the three-dimensioinal structure of the magnetic field in a spot can display a very complex topology from the photosphere to the chromosphere, with the simultaneous existence of regions of twisted field lines with opposite signs, or equivalently, the co-existence of regions of field with opposite helicity.

The magnetic field H in a sunspot correlates with the area, temperature, and brightness of the umbra: on average, the field is higher in darker, cooler, sunspots with larger areas. The dependence of H in sunspot umbrae on the spot area S was first obtained in [10]. Based on the results of several studies, it was concluded that the relationship between the maximum magnetic field B and the area S agrees with the empirical relation obtained by Houtgast and van Sluiters (see [1]), $B = 3700S/(S + 60)$, where B is measured in Gauss and S is measured in millionths of a hemisphere. The relation between B and S was studied using measurements of the vector magnetic fields in spots [11]. It was shown that there is a logarithmic dependence between the maximum field in the upper layers of a spot and the spot area. The relation between the magnetic field and the spot brightness is discussed in detail in [12].

The vast majority of the results described above concerning the properties of magnetic fields in sunspots were obtained either for single spots or for individual spots in sunspot groups, without considering the relationships between the studied spots and other spots in the group. In fact, spots with different properties can be distinguished in a sunspot group, and the group itself has certain characteristics determined by the collection of all the spots in the group. In most cases, the westernmost spot of a ground, which often has a large area, is called the leading sunspot. Spots in a group with the opposite field polarity are called trailing spots. The leading spot is usually closer to the equator than the trailing spots. According to the Hale law, in odd cycles, the magnetic fields of leading spots in a group in the Northern hemisphere have Northern polarity, while the fields of trailing spots have Southern polarity; this pattern changes sign in the Southern hemisphere and in the transition to even cycles [2].

However, there is relatively little evidence for differences in the properties of leading and trailing sunspots, either within a single group or averaged over many groups. There are no significant differences in the dependences of the contrasts [13] or photospheric magnetic fields [1] of leading and trailing spots on the spot area or on the stage of evolution of the spot.

It was shown in [14] that the dependence of the contrast in the HeII 304 Å line and the parameters of the HeI 10 830 Å infrared triplet on the areas of spots differ appreciably for leading and trailing spots. These results stimulated the current study, aimed at searching for the physical origin of this difference. We suppose that the properties of sunspot groups are a consequence of differences in the magnetic-field geometries above leading and trailing spots, which could also be manifest through asymmetry of the magnetic-field lines connecting the leading and trailing spots in an active region. To test this hypothesis, we have compared some properties of the magnetic fields in magnetically connected pairs of leading and trailing sunspots. We have also estimated the field properties in single sunspots for comparison.

2. DATA AND METHODS

Our study of the magnetic properties of sunspots has relied on calculations of the magnetic field in the solar atmosphere carried out using a potential field—source surface model. We have used this model in a potential approximation to calculate the three components of the magnetic field in the space betwen the photosphere and the source surface—a sphere with a radius equal to 2.5 times the radius of the Sun $R_\circ$. The field calculations were carried out using data from SOLIS (NSO) [15], which have a spatial resolution of 1″. We used the B_d technique [16], which makes it possible to obtain an "instantaneous" distribution of the magnetic field above the visible surface of the Sun, i.e., averaged only over the time required to take the magnetogram. The essence of this technique is the following. The magnetic field in the region between the photosphere ($R = R_\circ$) and the source surface ($R = R_S = 2.5R_\circ$) is assumed to be potential ($B(R) = -\nabla\psi$, where ψ is the scalar potential field) and to satisfy the Laplace equation, $\Delta\psi = 0$. Solving this equation requires specifying boundary conditions at $R = R_\circ$ and $R = R_S$. The boundary conditions for the Laplace equation should define the normal component of the magnetic field at both boundaries over the entire sphere. The problem is that only one hemisphere of the Sun is accessible to observation, and in that hemisphere only the component of the field along the line of sight is measured.

Before the appearance of the B_d technique, two approaches were used to specify the boundary conditions at the inner boundary of the computational region ($R = R_\circ$). In the first, a synoptic map was created of the measured field component along the line of sight B_l at each point on the surface at the time when a given point passed through the central meridian [17, 18], taking the field to be potential at the photospheric level. In the second approach, the field was not taken to be potential at the photospheric level, but it was assumed that the magnetic field is radial everywhere in the photosphere. In this case, the normal component of the field at the lower boundary can easily be calculated from observations of the longitudinal field together with a synoptic map [19].

In the B_d technique, the boundary conditions at the inner boundary of the computational region ($R = R_\circ$) are specified using the relations $-d\nabla\psi(R) = B_d(\theta,\varphi)$. Here, d is a unit vector directed along the line of sight toward the observer, B_d is the measured field component along the line of sight, and θ and φ are the latitude and longitude of the point at which the field is measured at $R = R_\circ$. The potential condition is retained at the inner boundary. To determine boundary conditions over the entire solar surface, it is sufficient to have two magnetograms, which provide the distribution $B_d(\theta,\varphi)$ on the visible surface of the Sun and the opposite side of the Sun. The values $B_d(\theta,\varphi)$ obtained for one measurement of the field distribution at the solar surface can be used as boundary conditions for the visible surface of the Sun.

The study [16] considered two approaches to formulating boundary conditions on the opposite side of the Sun. in the first, a magnetogram of the opposite side of the Sun is formed from the B_l component of the measured field (for example, a fragment of a synoptic map), and in the second, a magnetogram is formed from B_d synoptic maps obtained from a given set of daily magnetograms. The boundary condition at the source surface ($R = R_S$) in magnetic-field computations using the B_d technique are the same as in "classical" field computations using synoptic maps of the B_l field component: $\psi(R_S) = 0$. In other words, the magnetic-field lines at the source surface are taken to be radial.

To solve the Laplace equation with these boundary conditions, the potential field is represented as an expansion in spherical functions in the form of a finite series with L terms. It was shown in [16] that the Laplace equation has a unique solution in this case.

When computing the magnetic field using the SOLIS data, we first averaged the measured field over areas with linear sizes $\sim1^\circ$. This means that the minimum spatial resolution at the surface was $\sim16.7''$.

At the interface between the daily magnetograms on the visible surface of the Sun and the magnetograms formed in some way on the opposite side (i.e., at the limb), there should be some difference between the two magnetograms. This can lead to erroneous computations of the field near the limb. Such distortions are appreciable at the solar surface for computations with a large number of harmonics, and decrease as the number of harmonics decreases. This type of error also decreases when B_d synoptic maps are used as the boundary conditions on the opposite side of the Sun.

In our magnetic-field computations, we used an expansion of the potential field in 90 spherical-function harmonics. This corresponds to a spatial resolution of about $33.5''$ at the solar surface. Our selection of sunspots for the analysis was carried out using data from the Solar Dynamic Observatory (SDO) [20]. We analyzed continuum images of spots obtained in 2010–2013 with the Helioseismic and Magnetic Imager (HMI) [21] on board SDO. We selected 31 magnetically connected pairs of leading and trailing spots for the analysis (from 62 analyzed sunspot groups) located near the central meridian and at low latitudes $\leq 40^\circ$; i.e., near the center of the visible solar disk. The active regions harbouring the selected spots displayed predominantly α and β magnetic configurations.

We take "magnetically connected" spots to be pairs of spots whose umbrae were connected by the magnetic-field lines derived in the computations, or for which the Eastern end of a field line from the leading spot was close to the trailing spot and the Western end of a field line from the trailing spot was close to the leading spot. In two cases when a sunspot group was formed of only two spots, not considering pores, we found no magnetic connection between these spots. The absence of a magnetic connection between the leaading and trailing spots in these cases is probably the result of the relatively low spatial resolution of the magnetic-field computations. Finally, we also selected for comparison 15 single spots for which the bases of the Eastern ends of the field lines were located further from the equator than the spots themselves, thus imitating the absent trailing spots.

The table lists all the magnetically connected and single spots that were analysed together with the number of the active region in which they were located, the magnetic configuration of the spots in the active region, and the areas of the spots.

The following parameters were determined in the umbrae of all the selected spots:

(a) the minimum angle $a_{\min}$ between the magnetic-field lines and the radial direction,

(b) the magnetic field $B_{a-\min}$ at the point where $a = a_{\min}$,

(c) the area of the sunspot umbra S in millionths of a solar hemisphere (msh), Studied magnetically connected pairs of leading and trailing sunspots

Date	NOAA	MC	S_l, msh	S_f, msh	Date	NOAA	MC	S_l, msh	S_f, msh
1	2	3	4	5	1	2	3	4	5
02.07.2009	11084	α	23	–	08.10.2011	11309	β	18	–
13.07.2010	11087	β	11	2	15.10.2011	11316	$\beta\gamma$	25	25
03.08.2010	11092	α	42	–		11314	β	50	–
09.08.2010	11093	α	25	–	21.10.2011	11328	$\beta\gamma$	5	5
17.09.2010	11106	β	8	–		11327	$\beta\gamma$	38	31
20.10.2010	11115	α	25	–	28.10.2011	11330	$\beta\gamma$	67	40
08.12.2010	11131	α	73	–	10.11.2011	11339	$\beta\gamma\delta$	51	8
05.01.2010	11140	α	28	–		11331	$\beta\gamma\delta$	21	4
14.02.2011	11157	β	6	5	28.11.2011	11358	β	21	6
	11158	β	31	25	13.06.2012	11504	$\beta\gamma$	52	14
13.04.2011	11190	β	24	18		11505	$\beta\gamma$	5	8
05.05.2011	11205	β	16	5	28.06.2012	11512	β	36	8
22.05.2011	11216	α	8	–	02.07.2012	11513	$\beta\gamma$	39	7
07.06.2011	11232	α	12	–		11515	$\beta\gamma$	26	24
14.06.2011	11234	β	13	1		11517	$\beta\gamma$	21	6
29.06.2011	11242	β	4	6	10.08.2012	11543	$\beta\gamma$	36	10
11.07.2011	11249	α	8	–	01.01.2013	11640	β	30	3
	11245	α	6	–		11639	β	5	–
17.07.2011	11248	α	18	–	14.01.2013	11654	$\beta\gamma$	63	22
19.07.2011	11254	β	6	6		11656	$\beta\gamma$	6	2
03.08.2011	11158	$\beta\gamma\delta$	88	34	07.02.2013	11667	β	13	10
06.08.2011	11263	$\beta\gamma\delta$	47	6	15.07.2013	11791	$\beta\gamma$	2	13
	11267	$\beta\gamma\delta$	9	7	08.07.2013	11793	$\beta\gamma\delta$	29	11

The columns contain (1) the observation date, (2) the NOAA number of the active region, (3) the magnetic configuration (MC) of the active region, (4) the area of the leading spot S_l in millionths of a solar hemisphere, and (5) the area of the trailing spot S_f in millionths of a solar hemisphere. If column (5) contains a dash rather than a number, this indicates that this was a single spot.

(d) the length L_l of the portion of the field line from the leading spot to the place where the radial field component becomes zero, and the length from that location to the Eastern end of the field line L_f.

The angle a was found from the relation $a = \arccos(|B_r|/B)$, where B_r is the radial component of the photospheric magnetic field in heliographic coordinates and B is the magnetic-field strength at the measurement point.

3. RESULTS

Figure 1 presents typical examples of the studied sunspot groups and single spots. Fig. 1a depicts a group of magnetically connected spots in the active region NOAA 11242 observed on June 26, 2011. A simple β magnetic configuration is characteristic of this group of spots. The sunspots in NOAA 11242 have relatively modest areas with well defined umbrae and penumbrae. The leading spot of the group is located to the West of the trailing spot, and at lower latitude. The polarity of the leading spot obeys the Hale law. Figure 1b shows an example of a fairly complex sunspot group, in the active region NOAA 11654, which was observed on the solar disk on January 14, 2013. Both magnetically connected and unconnected spots can be seen. The magnetic

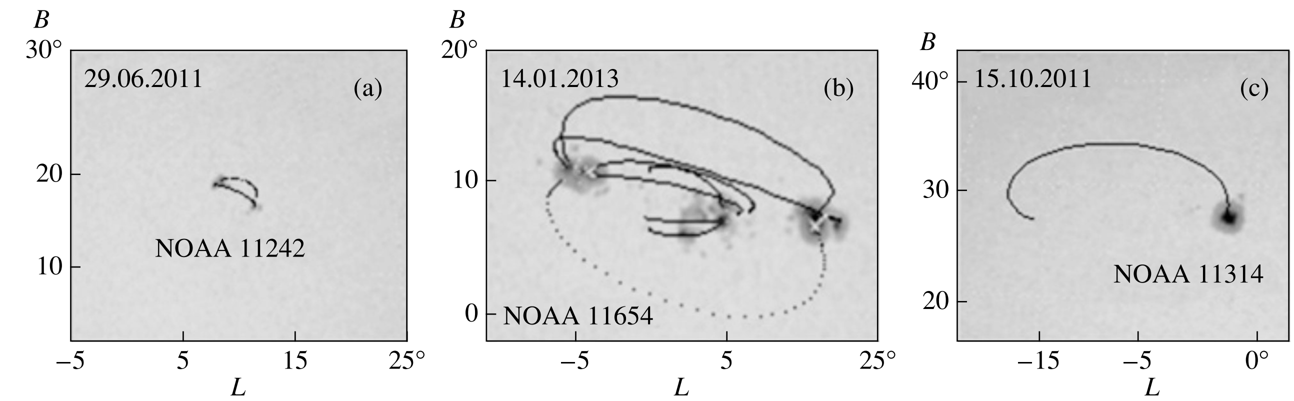

Fig. 1. Examples of spots analyzed: (a, b) sunspot groups and (c) a single sunspot.

configuration of the group is $\beta\gamma$. The Western spot with the largest area is the leading spot of the group, and is connected with the Easternmost trailing spot. However, spots with different polarities that are not connected to other spots in the group by magnetic-field lines can be seen at the center of the group. Figure 1c shows an example of a single spot, in the active region NOAA 11314, observed on October 15, 2011; the magnetic configuration of this active region with its single spot of leading polarity and a set of small pores was $\beta\gamma$.

Analysis of the magnetic-field structure in the umbra of each spot using the field computations in a potential approximation enabled us to determine the minimum angle $a_{\min}$. We found that, for ∼81% of the spot pairs considered, the minimum angle between the magnetic-field lines and the radial direction in the umbra was smaller in the leading spot than in the trailing spot ($a_{l-\min} < a_{f-\min}$). For spot pairs for which this is true, the mean minimum angle in the leading spot is $\langle a_{l-\min}\rangle = 14.8°$, while the mean angle in the trailing spot is $\langle a_{f-\min}\rangle = 26.44°$, roughly twice the angle for the leading spots. This is also reflected in the mean value of the ratio $\langle a_{l-\min}/a_{f-\min}\rangle = 0.576$. We suppose that the values of $a_{l-\min}$ and $a_{f-\min}$ in spots could be less than the values we have obtained, since the magnetic field is averaged over the solar surface in our computations, reducing the effective spatial resolution.

Figure 2a presents the dependence of $a_{l-\min}$ on $a_{f-\min}$ for pairs of spots with $a_{l-\min} < a_{f-\min}$. This figure shows that, on average, an increase in $a_{f-\min}$ is accompanied by an increase in $a_{l-\min}$; i.e., there is a positive correlation between the minimum angles in the leading and trailing spots. A linear fit gives the equation $a_{l-\min} = -0.4 + 0.58a_{f-\min}$ with a coefficient of determination $R^2 = 0.877$. Here and below, we use the coefficient of determination as a measure of the agreement between parameters compared; this is the ratio of the sum of the squared residuals relative to the linear fit divided by the total sum of the squared deviations from the mean. The dependent variable is predicted using a function of the explanatory variables, and the case at hand, a linear dependence, the coefficient of determination is equal to the square of the correlation coefficient. It is convenient to use the coefficient of determination in practice, since it is close to unity when the model has high significance and close to zero when it has low significance.

Let us now consider the relationship between $a_{l-\min}$ and $a_{f-\min}$ and the magnetic field values B_l and B_f at the points $a_{l,f-\min}$, where $a_{l,f-\min}$ denotes the mininum angle between the field line in the sunspot umbra and the radial direction for the leading (l) or trailing (f) spot (Fig. 2b). The field values B_l and B_f were coincident with or close to the maximum field in the sunspot umbra $B_{\max}$. For the leading spots, $a_{l-\min} = 21.8{-}0.02B_{l,f}$, with a coefficient of determination $R^2 = 0.33$; for the trailing spots, $a_{f_{\min}} = 38.4{-}0.04B_{l,f}$, with $R^2 = 0.43$. For our sample with $a_{l-\min} < a_{f-\min}$, we found the mean values $\langle B_l\rangle = 356.6$ G and $\langle B_f\rangle = 300.9$ G, so that $\langle B_l\rangle$ exceeds $\langle B_f\rangle$; here, the angular brackets $\langle\ldots\rangle$ denote an average over the sample. These sunspot field values are appreciably lower than the characteristic magnetic fields in even small pores [1, 2]. The reason for this is that virtually all the sunspot magnetic-field measurements were carried out with relatively high spatial resolution.

In our study, the values of B_l and B_f at the photospheric level were found from computations based on the mean measured fields, effectively reducing the spatial resolution. As was noted above, the field computations were carried out by expanding the field potential in a series in spherical functions with $L =$ 90 harmonics. This expansion provides a spatial resolution at the photospheric level of 33.5″, which

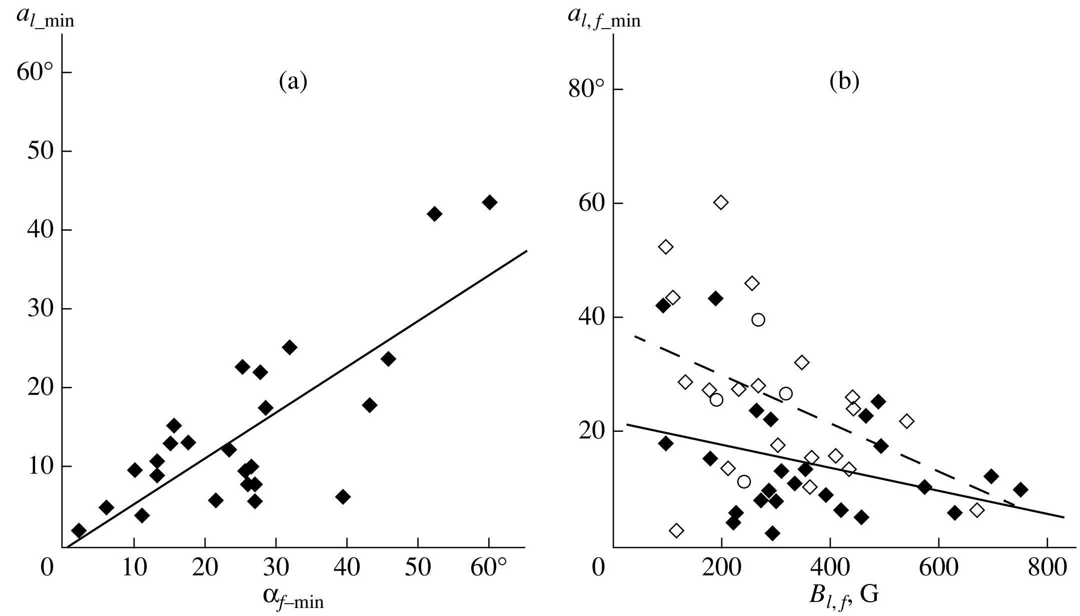

Fig. 2. (a) Dependence of $a_{l\_\min}$ on $a_{f\_\min}$ for pairs of spots with $a_{l\_\min} < a_{f\_\min}$. (b) Dependences of $a_{l,f\_\min}$ and $B_{l,f}$ separately for leading (filled symbols) and trailing (hollow symbols) spots.

is comparable to the sizes of the largest sunspots, and appreciably larger than the sizes of the more frequently observed modest sunspots with sizes not exceeding 10″.

It follows from Fig. 2b that there is a weak negative correlation between $a_{l,f\_\min}$ and $B_{l,f}$: on average, $a_{l,f\_\min}$ decreases with increasing $B_{l,f}$. In other words, the field lines are more radial in magnetic tubes forming the umbrae of both leading and trailing spots that have stronger fields at the photospheric level. Although this correlation is fairly weak, the relationship between the field strength and the deviation of the field direction from the normal is expressed fairly clearly: the stronger the magnetic field in a flux tube, the smaller the deviation of the field lines from the normal to the photospheric surface. Note also the following feature in Fig. 2b: for the same magnetic fields in the leading and trailing spots, on average, $a_{l\_\min} < a_{f\_\min}$. This means that, in addition to the strength and direction of the magnetic field, the values of $a_{l\_\min}$ and $a_{f\_\min}$ are also determined by other independent characteristics of the spots. Finally, it follows from Fig. 2b that the values of $a_{l\_\min}$ and $a_{f\_\min}$ in the leading and trailing spots with the highest fields are similar.

Figure 3 can be used to estimate the dependence of $a_{l\_\min}$ and $a_{f\_\min}$ on the area of the sunspot umbra. The mean area of the umbrae of the leading spots S_l was 23.2 msh, while the corresponding area for the trailing spots, S_f, was 9.45 msh. Figure 3 shows a weak negative correlation between $a_{l\_\min}$ and S_l, and also between $a_{f\_\min}$ and S_f. A linear fit to the data for the leading spots yields $a_{l\_\min} = 13.7 - 0.04S_{l,f}$, while the corresponding fit for the trailing spots yields $a_{f=min} = 28.6 - 0.37S_{l,f}$. This type of dependence is quite expected, given the known relationship between the maximum magnetic field in a spot and the spot area obtained by Houtgast and van Sluiters [1], and

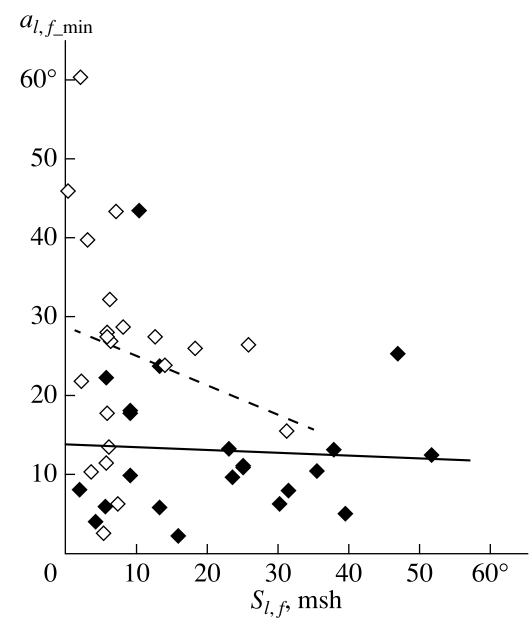

Fig. 3. Dependence of $a_{l,f\_\min}$ on $S_{l,f}$ separately for the leading (filled symbols) and trailing (hollow symbols) spots.

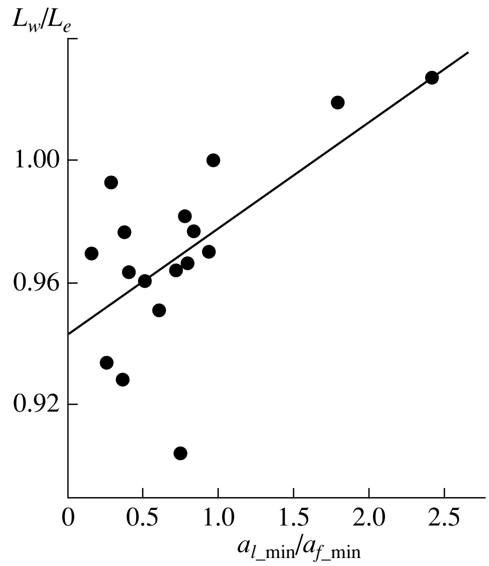

Fig. 4. Relationship between the ratios $a_{l-\min}/a_{f-\min}$ and L_w/L_e.

the correlation between $a_{l,f-\min}$ and $B_{l,f}$ in Fig. 2b. At the same time, we have detected a difference in the behavior of $a_{\min}$ as a function of the area $S_{l,f}$ considered separately for leading and trailing spots. In other words, the larger the area of the spot umbra, the more vertical the field in the spot, but with details of the relationships being different for leading and trailing spots. It also follows from Fig. 3 that, for equal umbra sizes, the minimum angle between the field lines and the radial direction at the bases of the field lines is, on average, smaller in leading spots than in trailing spots. Finally, in the umbrae with the largest sizes, on average, $a_{l-\min} \approx a_{f-\min}$.

Thus, Figs. 2b and 3 show that the inclination of the magnetic field in the umbrae of both leading and trailing spots depends on the umbra's field strength and size.

We believe that the difference between $a_{l-\min}$ and $a_{f_{\min}}$ reflects properties of the magnetic-field lines connecting the leading and trailing spots. The extent of this difference, which we have characterized using the ratio $a_{l-\min}/a_{f-\min}$, is manifest in asymmetry of the field lines. Since the umbrae of the leading and trailing spots in our sample are not all connected by field lines [for example, in some cases, the field lines emerging from a leading (or trailing) spot end up near a trailing (or leading) spot], we analyzed the asymmetry of the field lines emerging from the umbrae of leading spots only.

We used the ratio L_w/L_e of the length of the field line from the Western base in the leading spot to the place where the radial field component becomes equal to zero L_w and the distance from that location to the Eastern base of the field line L_e. We compared the ratio $a_{l-\min}/a_{f-\min}$ with L_w/L_e, jointly considering spots for which ($a_{l-\min} < a_{f-\min}$, $L_w < L_e$) and those for which ($a_{l-\min} > a_{f-\min}$, $L_w > L_e$). This relation is shown in Fig. 4; a linear fit to these data yields the equation $y = 0.94 - 0.04x$ with a coefficient of determination $R^2 = 0.65$. On average, the asymmetry of the field lines grows with increasing $a_{l-\min}/a_{f-\min}$. However, the asymmetry of the field line is not a very effective measure of the difference between $a_{l-\min}$ and $a_{f-\min}$, since the conditions $a_{l-\min} < a_{f-\min}$ and $L_w < L_e$, or $a_{l-\min} > a_{f-\min}$ and $L_w > L_e$, are jointly satisifed in only ∼60% of cases.

We also studied the magnetic properties of 15 single spots detected at the same epochs as those for which we carried out the analysis of bipolar pairs of leading and trailing spots. We selected spots for which the field lines emerging from their umbrae ended at a location that was to the East and further from the equator than the umbra itself. In this way, we created an analogy between the Eastern bases of the field lines emerging from the single spots and the trailing spots. An example of such a spot is shown in Fig. 1c.

Our analysis shows that the magnetic properties of single spots are, in many ways, the same as those of the leading spots in bipolar groups. The minimum angle between the field lines and the radial direction is about a factor of two smaller than this angle at the Eastern bases of the field lines emerging from these spots (9.7° and 20.9°, respectively), and the magnetic fields are higher (301.4 and 132.4 G). The magnetic field in a sunspot umbra is essentially independent of the minimum angle between the field lines and the radial direction; the coefficient of determination is 0.043, which indicates an absence of any correlation. Finally, there is a positive correlation between the ratios $a_{l-\min}/a_{e-\min}$ and L_w/L_e for spots selected using the same criteria as those applied for our selection of bipolar groups,where $a_{e-\min}$ is the minimum angle between the field lines and the radial direction at the Eastern end of the field lines emerging from the spot.

Thus, the magnetic properties of single spots are analogous to those we have identified for the leading spots of well developed spot groups.

We wish to turn your attention to the following result. It follows from Figs. 1a and 3a that $a_{l-\min}$ or $a_{f-\min}$ are 30° or more in several cases; this is particularly true of the trailing spots. Such values of $a_{\min}$ were also detected in some single spots. Such large minimum angles between the field lines and the radial direction, or nearly equivalently, between the spot axis and the radial direction, are overestimated compared to the real angles. The basis for this conclusion is the impossibility of observing such spots far from the central meridian, demonstrated by estimates made taking into account the thickness of the photosphere. At the same time, some spots with such $a_{\min}$ values are clearly observed near the limb. We propose that the appearance of large angles $a_{\min}$ is a consequence of the averaging of the calculated field over a relatively large spatial scale. Moreover, it may turn out that a potential approximation describes the magnetic-field configurations in some spot umbrae poorly.

4. CONCLUSION

We have compared the magnetic properties of bipolar sunspot groups containing of magnetically connected leading and trailing spots, and also studied the magnetic characteristics of a sample of single spots observed during 2010–2013.

In 81% of the bipolar groups studied, the minimum angle between the magnetic-field lines in the sunspot umbra and the radial direction is smaller in the leading than in the trailing spot. The mean ratio of these angles is $a_{l-\min}/a_{f-\min}{\sim}0.576$. There is a positive correlation between $a_{l-\min}$ and $a_{f-\min}$.

On average, the magnetic field at the point with the minimum angle between the magnetic-field lines and the radial direction $a_{\min}$ is higher in leading than in trailing spots. There is a weak negative correlation between $a_{l-\min}$ and the field B_l in the leading spot, and also between $a_{f-\min}$ and B_f in the trailing spot.

There is a modest negative correlation between the area of the leading spot and $a_{l-\min}$, and also between the area of the trailing spot and $a_{f-\min}$.

We have found a positive correlation between $a_{l-\min}/a_{f-\min}$ and L_w/L_e, where L_w and L_e are the lengths of the Western and Eastern parts of the field lines from the leading spot to the place where the radial field component becomes zero. This relation was found for pairs of spots with certain properties (see Section 3). The relationship between the parameters of the field lines issuing from the bases of the umbrae of single spots and the parameters of the same field lines at their Eastern bases is similar to the corresponding relationship for the field lines connecting leading and trailing spots.

Our results indicate that the field lines in the umbrae of spots with stronger magnetic fields are more radial. The field lines in the umbrae of leading spots are closer to radial than the field lines in the umbrae of trailing spots. In both leading and trailing spots, the deviation of the field lines from the radial direction decreases with increasing area of the spot umbra. Thus, we conclude that the inclination of the field lines to the radial direction in a sunspot umbra depends on the magnetic-field strength, transverse size of the magnetic tube, and the type of spot, i.e., whether it is a leading or trailing spot.

There could be various reasons for the dependence of the inclination of the magnetic-field lines to the radial direction on the field strength in the spot umbra and the type of spot (leading or trailing). For example, because the magnetic fields in leading spots usually exceed those in the corresponding trailing spots, this suggests that magnetoplasma tubes with stronger fields are more stable against deviation from the vertical position. These deviations also bring about the average values of $a_{l-\min}$ and $a_{f-\min}$ we have found.

We also suggest that the differences in the dependence of two parameters of the line emission (the emission contrast in HeII 304 Å) and absorption (the equivalent width of the HeI 10830 Å IR triplet) in leading and trailing spots on their areas found in [14] is associated with the asymmetry of the magnetic-field lines of sunspot groups we have discovered here. The asymmetry of the magnetic-field lines in a sunspot group could lead to an observed increase in the optical depth of the layer of HeI atoms in the 2^3S state, and therefore a flux of λ304 Å UV radiation above the trailing spot, compared to the leading spot. This must be taken into account when constructing models for individual sunspots, as well as groups of spots and activity complexes.

When analyzing the data considered here, we have determined the angle between the field lines and the normal to the solar surface. Note that another angle is important physically—the angle between the field lines and the sunspot axis. Although these two angles are fairly close, they could display differences in analyses of variations of the field strength and the direction of the field lines, for example, as a spot traverses the solar disk. The question of the inclination of the spot axis to the normal (or the related inclination of the spot surface to the surface of the unperturbed photosphere) has a long history. The possible existence of such inclinations was first noted in [22] based on an analysis of the frequency of encountering spot groups on the Eastern and Western limbs and their total areas. This question was subsequently considered in many studies, which are discussed in the monograph [23]. However, the situation remains unclear. There is not clear indication of either the magnitude of this inclination or possible differences for leading and trailing spots. If this effect is significant, it could be manifest as a group passes across the solar disk.

Note that all information about possible inclination of the spot axis have been obtained using indirect data obtained nearly 30 years ago, and only now has it become possible to verify hypotheses using direct observations with high resolution. The shape of a field line to some extent reflects the shape of the magnetic tube connecting the leading and trailing spots. The importance of this question is obvious, since a magnetic tube will inevitably be deformed by surface flows as it rises.

In our subsequent studies concerning the properties of the magnetic fields in sunspots, we plan to study the variations in the magnetic-field properties for leading and trailing spots as they pass across the solar disk. We also plan to use high-spatial-resolution vector magnetic-field measurements obtained with SDO/MHI.

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