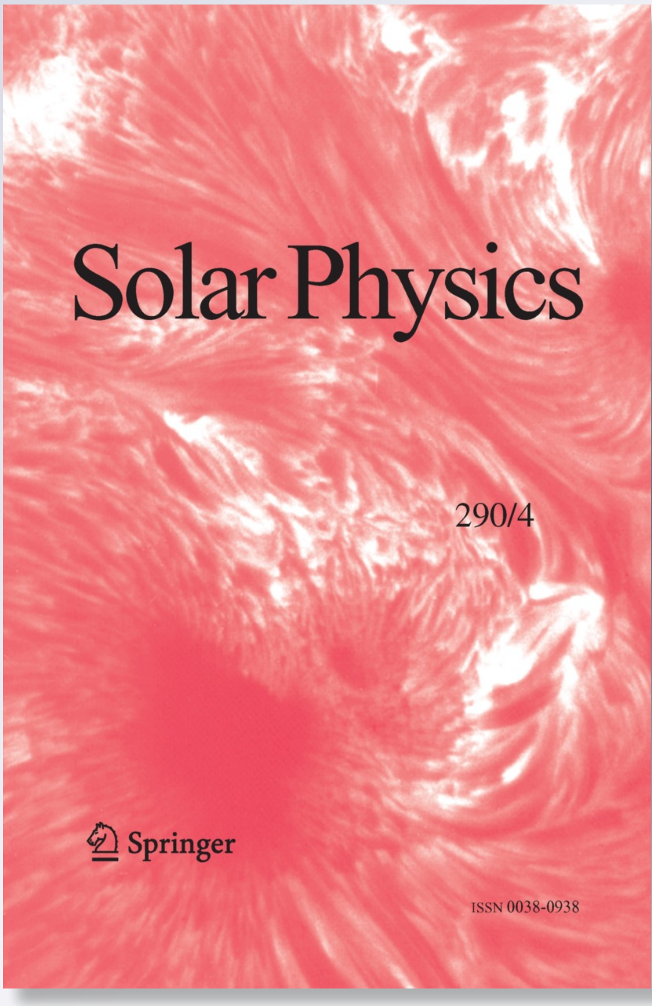
The Upper Limit of Sunspot Activity as Observed over a Long Time Interval

Yu. A. Nagovitsyn, V. N. Obridko & A. I. Kuleshova

Solar Physics
A Journal for Solar and Solar-Stellar Research and the Study of Solar Terrestrial Physics

ISSN 0038-0938
Volume 290
Number 4

Sol Phys (2015) 290:1285-1294
DOI 10.1007/s11207-015-0657-4



Springer

Solar Phys (2015) 290:1285–1294
DOI 10.1007/s11207-015-0657-4

The Upper Limit of Sunspot Activity as Observed over a Long Time Interval

Yu.A. Nagovitsyn[1] · V.N. Obridko[2] · A.I. Kuleshova[1]

Received: 16 June 2014 / Accepted: 2 February 2015 / Published online: 14 February 2015

Abstract After analyzing the observational manifestations of the α- and ω-effects of the dynamo theory and using the modified Waldmeier rule, we show that the annual mean Wolf numbers at the maximum of the 11-year cycle that are likely to occur a time interval of 10^4 years have an upper limit amounting approximately to $W_{\mathrm{EXTR}} \sim 230-240$. Similar values were also obtained using the results by Usoskin *et al.* (2014, Astron. Astrophys. 562, L10), who considered the probability of various activity levels by reconstructing the variations of solar activity over three thousand years. As an additional result, the predicted maximum of Cycle 24 is refined and is shown to be $W_{\mathrm{M}} = 72-132$ with a 95 % confidence.

1. Introduction

Routine observations of solar activity have been carried out since the 1820s. Individual instrumental records are available since 1610 (the time when the telescopes came into use for astronomical observations). There are also some proxy data that allow us to asses more or less reliably the level of solar activity over the Current Era (Ogurtsov *et al.*, 2002), in the epoch of Holocene (Usoskin, 2013), and even over the past 40 000 years (Ogurtsov, 2010).

Fundamentally, the question is how typical the extreme solar activity values obtained over a fairly short period of regular solar monitoring are and how much they might change over longer time scales, *e.g.*, 10 000 years.

Observations of solar-type stars of late spectral classes show that on the one hand, the energy released in flares can be much higher than observed in the Sun (Zeleny and Veselovsky, 2008). On the other hand, we do not know whether the specific effective parameters of a

✉ Yu.A. Nagovitsyn
nag@gao.spb.ru

1 Central Astronomic Observatory at Pulkovo of the Russian Academy of Sciences, Pulkovskoe Shosse 65, St.-Petersburg, 196140, Russia

2 Pushkov Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation of the Russian Academy of Sciences, Troitsk, Moscow, 142190, Russia

given star (magnetic field, rotation, development of convection) strictly determine its energy release limit.

The statistics of occurrence of grand minima and maxima of solar activity was studied by Usoskin, Solanki, and Kovaltsov (2007). In this article, we investigate whether the solar activity level can be arbitrarily high or if it has a certain upper limit. This is important because extremely high solar activity can influence both space weather as a whole and its terrestrial manifestations. In addition, this question is of fundamental interest to the dynamo theory.

This article provides evidence that the amplitude of the 11-year cycle of solar activity has a certain upper limit. In addition, we discuss the probable amplitude of the current Cycle 24.

2. Dynamo Theory and the Upper Limit of the Cycle Amplitude

The recent theory of the generation and variation of the solar magnetic field-$\alpha\omega$-dynamo is based on the combined action of two processes. The ω-effect arising as a result of differential rotation of the Sun transforms the global poloidal magnetic configuration to a toroidal one. The α-effect, in contrast, transforms toroidal fields into poloidal ones. This effect is usually associated with small-scale cyclonic flows. There is also a particular type of the α-effect due to the emergence of toroidal fields – the so-called Babcock–Leighton mechanism. Olemskoy, Choudhuri, and Kitchatinov (2013) explained the existence of the global activity minima and maxima by fluctuations of the α-effect. We consider both effects of the dynamo theory using experimental data. As an index of the global toroidal field we took the annual mean Wolf numbers, W, (http://sidc.oma.be/html/sunspot.html) and as an index of the poloidal or (more precisely) large-scale field, the annual mean dipole–octupole A-index (Tlatov and Makarov, 2005), which varies in antiphase with W (*i.e.*, is highest at the minimum of the cycle).

We consider the ω-effect first. For this purpose, we compare $A(t)$ with $W(t+\Delta)$, *i.e.*, with the sunspot activity delayed by Δ years. The delay time is chosen such that it ensures the best correlation between the indices for the form $W(t+\Delta) = bA(t)$. Figure 1a illustrates the dependence plotted with the original $A(t)$ values from Tlatov and Makarov (2005), and Figure 1b shows the same dependence plotted with $A(t)$ reconstructed since 1700 from Nagovitsyn *et al.* (2008). The dashed lines show the 95 % confidence intervals. Their calculation for prediction purposes with the standard deviation, s, taken into account is described in detail in Section 3.

The relations represented in Figures 1a and b obtained by the least-squares method are, respectively,

$$W(t+\Delta) = (0.872 \pm 0.034)A(t), \qquad k = 0.799, \qquad s_W = 28.9, \tag{1}$$

$$W(t+\Delta) = (0.839 \pm 0.019)A(t), \qquad k = 0.820, \qquad s_W = 24.2, \tag{2}$$

where $\Delta = 6$ years and s_W is the standard deviation for the fitting curve. The correlation coefficient k equal to about 0.8 corroborates the existence of the ω-effect. For the chosen indices, it is linear; *i.e.*, no significant nonlinearity was found.

Now, we turn to the α-effect. Compare $W(t-\Delta)$ and $A(t)$, again, with the most appropriate delay time; Figure 1d for the values by Tlatov and Makarov (2005) and Figure 1e for the values by Nagovitsyn *et al.* (2008). The statistical dependencies are

$$\begin{aligned} &A(t) = (1.74 \pm 0.11)W(t-\Delta) - (6.20 \pm 0.88) \cdot 10^{-3} W^2(t-\Delta), \\ &k = 0.738, \qquad s_A = 29.7, \end{aligned} \tag{3}$$

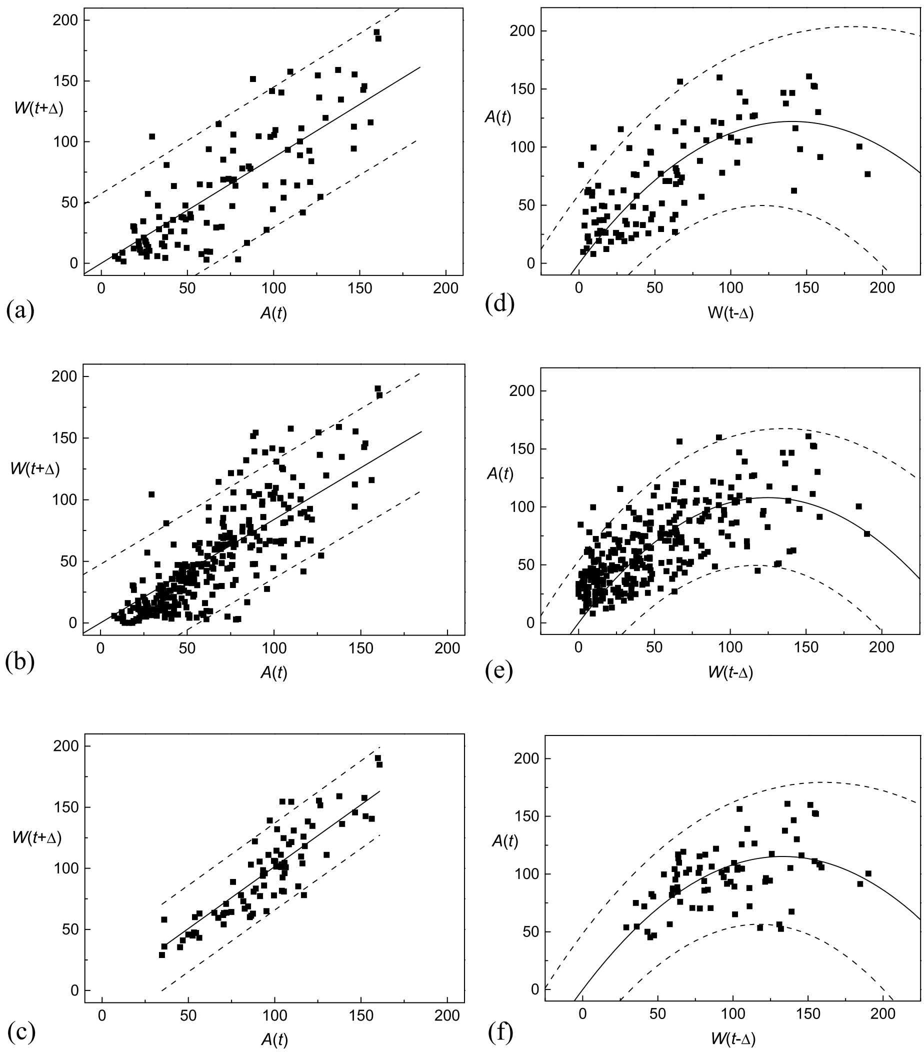


Figure 1 (a), (b) and (c): The Wolf number, W, as a function of the dipole–octupole, A, index in the past minimum. (a) Assuming a constant shift (Δ) as in Tlatov and Makarov (2005). (b) Idem (a) as in Nagovitsyn *et al.* (2008). (c) Considering the best shift as in Nagovitsyn *et al.* (2008), (d), (e) and (f): the A-index in function of W in the past maximum. (d) Assuming a constant Δ as in Tlatov and Makarov (2005). (e) Idem (d) as in Nagovitsyn *et al.* (2008). (f) Considering the best shift as in Nagovitsyn *et al.* (2008). Continuous lines emphasize regression curves, dashed curves correspond to 95 % confidence intervals.

$$\begin{aligned} &A(t) = (1.734 \pm 0.066) W(t - \Delta) - (6.96 \pm 0.58) \cdot 10^{-3} W^2(t - \Delta), \\ &k = 0.695, \qquad s_A = 26.8, \end{aligned} \tag{4}$$

where $\Delta = 5$ years. As expected, the sum of the delay times for both effects is 11 years. From now on, we recall that $W(t - \Delta)$ corresponds to the best forward shift with respect to $A(t)$, while $W(t + \Delta)$ corresponds to a backward shift. The shifts are different in each case.

It is important to note that the coefficients in Equations (1)–(2) and (3)–(4) are statistically compatible. This demonstrates the consistency of the reconstruction by Nagovitsyn *et al.* (2008) with experimental data. At the same time, since the number of points for Equations (2) and (4) is almost three times greater than for Equations (1) and (3), we use below the values of A from Nagovitsyn *et al.* (2008). Note that, although the correlation for the α-effect is lower than for the ω-effect, it is, nevertheless, high enough; the correlation coefficient is on the order of 0.7. The nonlinear character of the dependence $A(t) = cW(t-\Delta) + dW^2(t-\Delta)$ is clear as well, where c and d are some coefficients; the reliability of the negative coefficients of the quadratic terms in Figures 1d and 1e is 7σ and 12σ, respectively. These may be regarded as the leading terms in the Taylor expansion of the (unknown) nonlinear functional dependence of A on $W(t-\Delta)$. On the other hand, the dependence $W(t+\Delta)$ *vs.* $A(t)$ increases linearly.

We can write $A(t)$ as a function of $W(t-\Delta)$ as

$$A(t) \pm \sigma_A = \left[cW(t-\Delta) + dW^2(t-\Delta)\right] \pm \sqrt{\left[W(t-\Delta)\sigma_c\right]^2 + \left[W^2(t-\Delta)\sigma_d\right]^2 + s_A^2}$$

and $W(t+\Delta)$ as a function of $A(t)$ as

$$W(t+\Delta) \pm \sigma_W = bA(t) \pm \sqrt{\left[A(t)\sigma_b\right]^2 + [b\sigma_A]^2 + s_W^2}.$$

This will yield

$$\begin{aligned} W(t+\Delta) &\pm \sigma_W \\ &= \left[cbW(t-\Delta) + dbW^2(t-\Delta)\right] \\ &\quad \pm \sqrt{cW^2(t-\Delta)\sigma_b^2 + dW^4(t-\Delta)\sigma_b^2 + b^2W^2(t-\Delta)\sigma_c^2 + b^2W^4(t-\Delta)\sigma_d^2 + b^2s_A^2 + s_W^2}, \end{aligned} \tag{5}$$

where σ_A, σ_W and σ_c, σ_d are the standard deviations of the functions $A(t)$, $W(t+\Delta)$ and the coefficients c, d. The probability of exceeding the upper limit of the Wolf numbers in the Gaussian statistics is

$$P(W \geq W_{\mathrm{M}} + z\sigma_W) = 1 - \psi(z), \qquad \psi(z) = \sqrt{\frac{2}{\pi}} \int_0^z \mathrm{e}^{-\frac{t^2}{2}}\, \mathrm{d}t, \tag{6}$$

where $\psi(z)$ is the probability integral.

We assume that a virtually improbable event (VIE) is an event whose statistical occurrence rate is lower than once every 10 000 years. Since in Equations (1)–(4) we consider the annual values, the total number of the events in this case is $N = 10^4$. Obviously, for VIE, $1 - \psi(z) = \frac{1}{N} = 10^{-4}$. From Equation (6) we find that $z = 3.88$. By substituting the coefficients from Equations (2) and (4) to Equation (5), we plot $W(t+\Delta) + 3.88\sigma_W$ as a function of $W(t-\Delta)$, the open circles in Figure 2. Finding the maximum, we obtain that a VIE is $W > 226$. Rounding off, this yields in the case under consideration $W_{\mathrm{EXTR}} = 230$.

We have assumed in the previous calculations that the characteristic times of the α- and ω-effects are constant and are equal to five and six years, respectively. In fact, this assumption is too rigorous. It would be more convenient to consider the relations between the maxima of $A(t)$ and $W(t)$ in two successive activity cycles. For this purpose, the value of Δ is defined as the time delay between three consecutive maximal values of the $A(t)$ and $W(t)$ functions in their 11-year cycles. Obviously, the value of Δ will be different in each case.

Figures 1c and 1f represent the obtained dependencies expressed by the relations

$$W(t+\Delta) = (1.014 \pm 0.020)A(t), \qquad k = 0.88, \qquad s_W = 17.7 \tag{7}$$

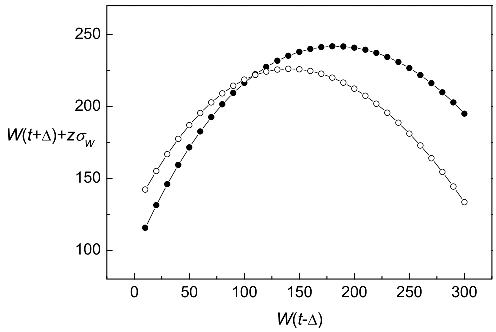


Figure 2 $W(t+\Delta)+3.88\sigma_W$ as a function of $W(t-\Delta)$ assuming that the characteristic times of the α- and ω-effects are constant (open circles), and $W(t+\Delta)+3.26\sigma_W$ as a function of $W(t-\Delta)$ assuming the best deviations for Δ at maximum $A(t)$ and $W(t)$ in the successive cycles (dark circles).

and

$$A(t) = (1.731 \pm 0.095) W(t-\Delta) - (6.50 \pm 0.75) \cdot 10^{-3} W(t-\Delta)^2, \quad k = 0.499, \qquad s_A = 24.2. \tag{8}$$

The assumption of choosing the best deviations for Δ at maximum $A(t)$ and $W(t)$ in the successive cycles increases the correlation coefficient for the ω-effect to 0.9, which indicates that the assumption is justified. Substituting the coefficient from (7) and (8) to (5), we obtain the curve that is shown in Figure 2 with filled circles. The total number of the considered events (11-year cycles) in our case is $N = \frac{10^4}{11} \approx 909$. Hence $\psi(z) = \frac{N-1}{N} \approx 0.9989$ and $z = 3.26$. Then, from Figure 2 it follows that $W_{\mathrm{EXTR}} = 240$.

3. Dynamo Theory and the Maximum Amplitude of the Cycle: Alternative Representations of the α-Effect

In Figure 1f, we can see that the points are highly spread. Therefore, there are alternative ways to represent $A(t)$ *vs.* $W(t-\Delta)$. First, we consider the linear form. Using the maximum values for three successive years as described above and applying the least-squares method, we obtain

$$A(t) = (0.935 \pm 0.037) W(t-\Delta), \qquad k = 0.469, \qquad s_A = 34.0. \tag{9}$$

By choosing two independent series of normally distributed random values and normalizing them to s_W, s_A, we can, with the aid of Equations (7) and (9), numerically simulate the variation of W over the time interval of 10^4 years. Figure 3a illustrates the desired variation of Wolf numbers for the linear dependence $A(t)$ *vs.* $W(t-\Delta)$. The variation of W is irregular with fluctuations on the order of the observed mean values. This behavior does not resemble the experimental reconstruction for the Holocene epoch (Solanki *et al.*, 2004). Thus, the linear dependence should be rejected as improbable.

Note that a similar numerical simulation for the parabolic dependence shown by Equation (8) discussed in Section 2 indicates that W is really stable, *i.e.*, it does not exceed a certain level during 10^4 years (see Figure 3b). The highest value reached for this period was $W_{\mathrm{EXTR}} = 230$. We consider other possible relations between $A(t)$ and $W(t-\Delta)$ describing the α-effect.

The α-effect may be determined by random fluctuations (Olemskoy, Choudhuri, and Kitchatinov, 2013). We can obtain with observations

$$A(t) = 95.3 \pm 29.2. \tag{10}$$

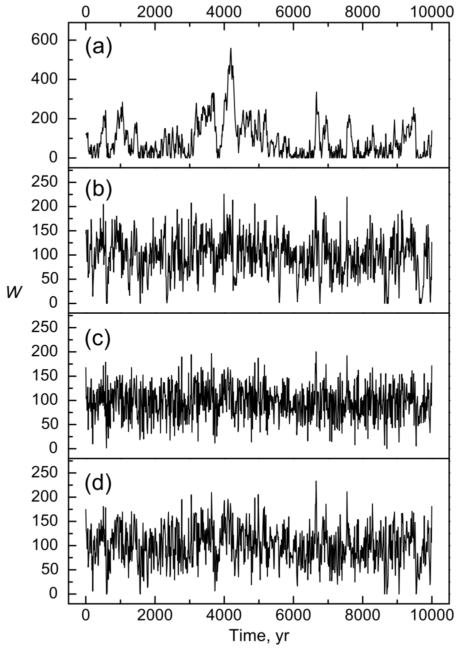


Figure 3 Numerical simulation of the Wolf number time variations using different assumptions on $A(t)$ *vs.* $W(t-\Delta)$: (a) linear dependence – Equation (9); (b) parabolic dependence – Equation (8); (c) random fluctuations of the α-effect – Equation (10); (d) power-law dependence – Equation (11).

The numerical simulation for this case gives the curve represented in Figure 3c. In this figure, the VIE limit is $W_{\mathrm{EXTR}} = 200$.

The final form alternative to Equation (8) that we considered is a power-law dependence $A(t) \sim W^m(t-\Delta)$, where $0 < m < 1$. Selecting m by the least-squares method, we obtain

$$A(t) = (10.17 \pm 0.55)\,W^{0.35}(t-\Delta). \tag{11}$$

The numerical simulations of Equations (7) and (11) yield the pattern shown in Figure 3d, with the VIE limit being $W_{\mathrm{EXTR}} = 230$.

The four plots in Figure 3 were obtained using two similar series of normally distributed random numbers, which facilitates comparing different dependencies $A(t)$ *vs.* $W(t-\Delta)$.

The choice of the best dependence for the α-effect is not our aim. It is relevant that all the dependencies considered above, except the linear one, show the limited nature of W.

The nonlinear, limited dependence $A(W)$ for the α-effect ensures a stable periodicity over large time scales, because if, for example, both dependencies were linear, the amplitudes of the successive cycles would increase indefinitely as soon as A exceeds a certain limiting value.

4. The Modified Waldmeier Rule and the Amplitude Limit of the Cycle

To begin with, we note that as shown in Nagovitsyn and Kuleshova (2012), the well-known Waldmeier rule (Waldmeier, 1935), which originally relates the height of the maximum of the 11-year cycle of solar activity, W_{M}, to the duration of its ascending branch, can be reformulated in terms of the solar activity growth rate after the preceding minimum. The highest growth rate of the annual mean of the Wolf numbers, $\dot{W}$, at the rise of the cycle is closely related to W_{M}: the correlation coefficient is $k = 0.95$ against $k \sim 0.83$ for various

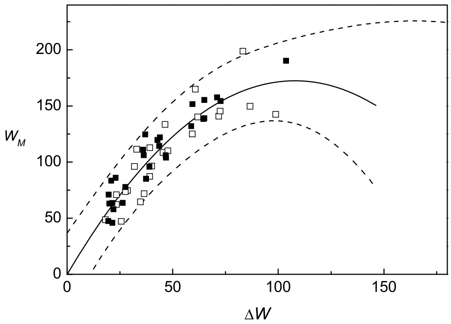


Figure 4 Equation (12) as a solid curve with the dashed prediction intervals and the observational annual means calculated both by the standard method from January to December (filled squares) and with a half-year shift from July to June of the following year (open squares).

modifications of the Waldmeier rule. The relationship between the maximum $\dot{W}$ and W_{M} was reported earlier in Dmitrieva, Kuzanyan, and Obridko (2000), but in this work, the authors used the smoothed monthly mean values and a linear (contrary to Nagovitsyn and Kuleshova, 2012) relation with nonzero W_{M} at $\dot{W} = 0$. A high correlation coefficient k allowed us to formulate in Nagovitsyn and Kuleshova (2012) a new prediction method and to estimate the height of maximum of Cycle 24 following an unusually long minimum (Obridko, Nagovitsyn, and Georgieva, 2012). It was determined that this cycle would have a medium height, the annual mean W_{M} value amounting to 104 ± 12 if the forecast proves correct.

Nagovitsyn and Kuleshova (2012) used the annual mean Wolf numbers, W_i, and $\dot{W}$ was estimated as $\Delta W = W_i - W_{i-1}$. The annual mean values were calculated by a standard method from the monthly means from January to December of the given year. Two years have passed since the article was published, and the maximum of the cycle is now closer. Therefore, we re-evaluate our conclusions.

The annual means need not necessarily be calculated in a standard way over a calendar year. We can use, for example, the values from July to June of the following year, *i.e.*, shift our calculations by half a year. This is what we did. As an approximation, we used a second-degree polynomial without the free term since, obviously, $W_{\mathrm{M}} = 0$ (cycle minimum) if the maximum $\dot{W} = 0$.

Figure 4 illustrates the relation $W_{\mathrm{M}} = f(\Delta W)$ plotted with the annual means calculated both by the standard method (filled squares) and with a half-year shift (open squares). In the second case, the correlation is somewhat lower, but it still remains very high, $k = 0.93$. The r.m.s. dependence determined from all points in the figure can be written as follows:

$$W_{\mathrm{M}} = (3.23 \pm 0.12)\Delta W - (1.52 \pm 0.18) \cdot 10^{-2} \Delta W^2, \qquad \sigma = 14. \tag{12}$$

Substituting in this equation the maximum value of ΔW in the rising phase of Cycle 24, which equals 39.2, we obtain the expected cycle maximum $W_{\mathrm{M}} = 102 \pm 14$ in agreement with Nagovitsyn and Kuleshova (2012).

Note that here as well as in Section 3, the negative sign before the quadratic term in Equation (11) means that the function is limited from above and cannot be arbitrary large. Since we are dealing with an extrapolation problem, the results obtained must be verified using the so-called prediction interval equal to the square root of the sum of the squared values of the confidence interval and the standard deviation of the experimental points from the fitting curve. Quantile 2 multiplied by this quantity yields an estimate at a 95 % confidence level.

The predicted amplitude of Cycle 24 obtained by this method is $P(72 < W_{\mathrm{M}} < 132) = 0.95$. Since the forecasting for Cycle 24 is difficult because of its unusual behavior (Obridko, Nagovitsyn, and Georgieva, 2012), the real value will, probably, be close to the lower limit.

The value of ΔW, at which W in Equation (12) is maximum, is $\Delta W = 106.3$. This is higher than all values recorded for the entire history of Wolf number observations (26 cycles). The really observed maximum $\Delta W = 103.7$. In this case, we can therefore only roughly estimate W_{EXTR}. The number of events with $\Delta W > 103.7$ recorded during the observation interval under consideration was no more than one per 27 cycles. If we assume this interval to be statistically significant, then the number of such events for 909 cycles (10^4 years) will be no more than $909/27 \approx 34$. Hence, $\psi(z) = \frac{N-1}{N} = 33/34 = 0.9706$, *i.e.*, $z = 2.18$. Then from Equation (12), we can estimate W_{EXTR} at $\Delta W > 103.7$:

$$W_{\mathrm{EXTR}} = 3.23\Delta W - 0.0152\Delta W^2 + 2.18\sqrt{(0.12\Delta W)^2 + \left(0.00018\Delta W^2\right)^2 + 14^2}.$$

From the values obtained, we chose the maximum, which is $W_{\mathrm{EXTR}} = 240$. Note again that since this value is reached for a ΔW that was never observed, the estimate obtained here is approximate.

5. Solar Activity During the Past Three Millenia and the Amplitude Limits of the Cycle

We estimate the probability of an extremely high solar activity discussed above by considering its variations over a long time interval. For this purpose, we used the reconstruction by Usoskin *et al.* (2014) based on the latest carbon cycle data, generation of ^{14}C, and variations in the geomagnetic dipole moment. The reconstruction covers a time interval of 3000 years.

First of all, note that the conclusion of two modes of solar activity corresponding to high and low activity made by Usoskin *et al.* (2014) agrees with the conclusion of Nagovitsyn, Pevtsov, and Livingston (2012) that large and small sunspots, whose populations form a bimodal distribution, are generated by different dynamo mechanisms.

Furthermore, Usoskin *et al.* (2014) provided the experimental probability density function for different values of solar activity found in the reconstruction (taking into account the uncertainties of the original models), as well as its approximation by two Gaussian curves. In this article, the approximation is used to estimate the probability of extremely high values of solar activity.

Usoskin *et al.* (2014) used in their reconstruction the Hoyt and Schatten (1998) Group Sunspot Numbers (G), while we are using the Wolf Sunspot Numbers (W). Although many authors do not separate these indices, we compared their annual mean values over a relatively trustworthy time interval from 1826 (the beginning of regular observations by Schwabe) to 1995 (the end of the Hoyt and Schatten G series). We obtain

$$W = (1.057 \pm 0.010)G, \qquad k = 0.979, \qquad s = 8.8,$$

which implies at 5.7σ that the indices under discussion are different.

The reconstruction by Usoskin *et al.* (2014) deals with 10-year means (G_{10}). We compare these values with the maximal decadal (G_{M}) in the period 1700 – 1995. We obtain

$$G_{\mathrm{M}} = (2.065 \pm 0.056)G_{10}, \qquad k = 0.958, \qquad s = 14.5.$$

This correlation is shown in Figure 5. In Usoskin *et al.* (2014) the probability density function (PDF) is plotted in the form of two normal distributions that represent the high and low G_{10} values. By digitizing the graph from Usoskin *et al.* (2014), we obtain for the former the mean $G_{10} = 44.4$ with a standard deviation $\sigma_{G_{10}} = 11.8$. By reducing the G_{10} scale to the Wolf number scale following statistical rules, we obtain for this distribution

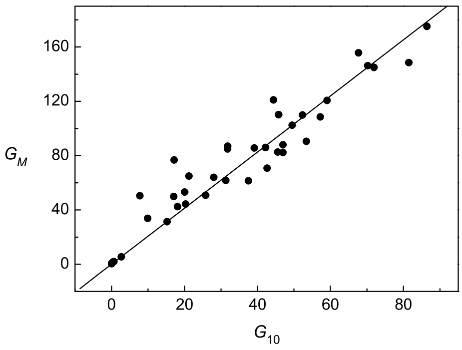


Figure 5 The group sunspot numbers: 10-year means, G_{10}, and their comparison with the maximal decadal, G_M, in the period 1700 – 1995 – the filled circles and the linear regression fit – the continuous curve.

$W = 97.0 \pm 31.4$, as above. It follows from the PDF plot in Usoskin *et al.* (2014) that high values account for 0.79 of the total PDF area, which corresponds to 790 decades. Using the same line of reasoning as above, we obtain $\psi(z) = \frac{N-1}{N} = 789/790 = 0.9987$, $z = 3.22$. Hence, the VIE upper limit in this case is $W > 198$. This estimate is somewhat lower than obtained in this article, but it should be noted that the PDF in Usoskin *et al.* (2014) involves negative G_{10}! If this implies that the general level of activity in the reconstruction is underestimated (by ~ 10 units if judged from the distribution), then $W_{\mathrm{EXTR}} = 220$.

6. Results and Conclusions

Considering the observed manifestations of the α- and ω-effects of the dynamo theory and using the modified Waldmeier rule, we have shown that the limiting value of the annual mean Wolf number at the maximum of the 11-year cycle that are likely to occur in the time interval 10^4 years does exist and is $W_{\mathrm{EXTR}} \sim 230 - 240$. We have considered a possible limitation of the solar activity level using various assumptions. The low value $W_{\mathrm{EXTR}} = 200$ corresponds to the hypothesis of random fluctuations of the α-effect. When considering the probability of high levels of solar activity by using a reconstruction of its variations over a time interval of 3000 years, we also obtained $W_{\mathrm{EXTR}} = 200$; however, we remark that this value could be higher ($W_{\mathrm{EXTR}} = 220$). Finally, a limiting value of $W_{\mathrm{EXTR}} \sim 230 - 240$, as the most probable one, is the main result of our work. Since in this article we discussed the upper limit of the activity cycle amplitude, we recall, to avoid misunderstandings, that we used the annual mean sunspot numbers. The smoothed monthly means can be 10 % higher (if they are not smoothed, they can be higher by 30 %).

The important feature of the processes under consideration is their nonlinearity. In the development of the cycle, the nonlinearity of the α-effect limits the amplitude of the cycle of the large-scale (polar) magnetic field from above, and during the emergence of a new flux, the (highest) rate of the emergence is related nonlinearly to the amplitude of the developing cycle.

An additional result of our work is the confirmation of our earlier forecast (Nagovitsyn and Kuleshova, 2012) of the Wolf number value at the maximum of Cycle 24. Taking into account the confidence interval of the approximating curve and the standard deviation, it is $W_M = 72 - 132$ with a 95 % reliability level, which is close to the estimates obtained by other authors (Petrovay, 2010; Kilcik *et al.*, 2009; Obridko and Shelting, 2009; Ozheredov, Breus, and Obridko, 2012). Note that the recent data for the first ten months of 2014 allow us to estimate the annual mean value for the secondary maximum of Cycle 24, which is expected not to exceed 80.

Acknowledgements The work was partly supported by the Russian Foundation for Basic Research, grant number 13-02-00277, and by the Programs No. 9 and 41 of the Presidium of the Russian Academy of Sciences.

References

Dmitrieva, I.V., Kuzanyan, K.M., Obridko, V.N.: 2000, *Solar Phys.* **195**, 209. DOI.
Hoyt, D.V., Schatten, K.H.: 1998, *Solar Phys.* **179**, 189. DOI.
Kilcik, A., Anderson, C.N.K., Rozelot, J.P., Ye, H., Sugihara, G., Ozguc, A.: 2009, *Astrophys. J.* **693**, 1173. DOI.
Nagovitsyn, Yu.A., Kuleshova, A.I.: 2012, *Astron. Rep.* **56**, 800. DOI.
Nagovitsyn, Yu.A., Pevtsov, A.A., Livingston, W.: 2012, *Astrophys. J. Lett.* **758**, L20. DOI.
Nagovitsyn, Yu.A., Miletsky, E.V., Ivanov, V.G., Guseva, S.A.: 2008, *Cosm. Res.* **46**, 283. DOI.
Obridko, V.N., Nagovitsyn, Yu.A., Georgieva, K.: 2012, *Astrophys. Space Sci. Proc.* **30**, 1. DOI.
Obridko, V.N., Shelting, B.D.: 2009, *Astron. Lett.* **35**, 247. DOI.
Ogurtsov, M.G.: 2010, *Geomagn. Aeron.* **50**, 475. DOI.
Ogurtsov, M.G., Nagovitsyn, Yu.A., Kocharov, G.E., Jungner, H.: 2002, *Solar Phys.* **211**, 371. DOI.
Ozheredov, V.A., Breus, T.K., Obridko, V.N.: 2012, *Izv., Atmos. Ocean. Phys.* **48**, 711. DOI.
Olemskoy, S.V., Choudhuri, A.R., Kitchatinov, L.L.: 2013, *Astron. Rep.* **90**, 458. DOI.
Petrovay, K.: 2010, *Living Rev. Solar Phys.* **7**, 1. DOI.
Solanki, S.K., Usoskin, I.G., Kromer, B., Schüssler, M., Beer, J.: 2004, *Nature* **431**, 1084. DOI.
Tlatov, A.G., Makarov, V.I.: 2005, *ASP Conf. Ser.* **346**, 415.
Usoskin, I.G.: 2013, *Living Rev. Solar Phys.* **10**, 1. DOI.
Usoskin, I.G., Solanki, S.K., Kovaltsov, G.A.: 2007, *Astron. Astrophys.* **471**, 301. DOI.
Usoskin, I.G., Hulot, G., Gallet, Y., Roth, R., Licht, A., Joos, F., Kovaltsov, G.A., Thébault, E., Khokhlov, A.: 2014, *Astron. Astrophys.* **562**, L10. DOI.
Waldmeier, M.: 1935, In: *Astron. Mitt. Eidgen* **133**, Sternw, Zürich, 1.
Zeleny, L.M., Veselovsky, I.S.: 2008, In: *Plasma Heliogeophysics (Plazmennaya Geliogeofizika)* **2**, Fizmatlit, Moscow.